Solutions for module 7

Markov chain Monte Carlo methods

Part 1

$$a(x, y) = \min\{1, \exp(-\frac{1}{2}(y^2 - x^2))\}.$$

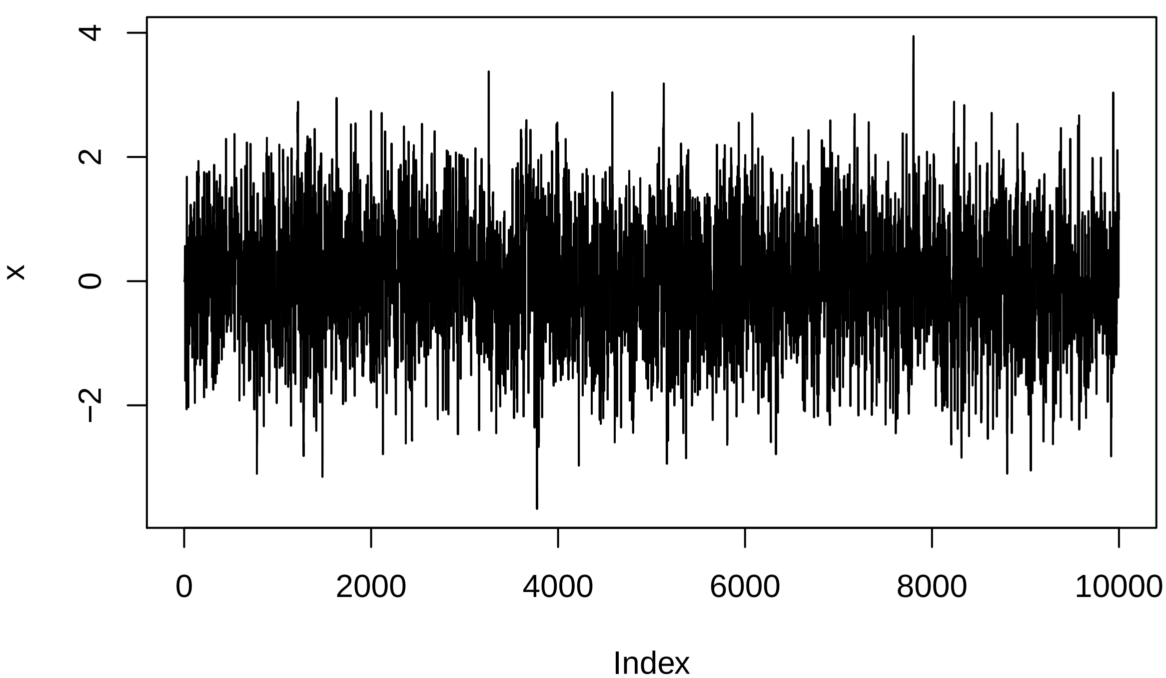
Part 2

Since u is uniformly distributed on $[0, 1]$, the probability that $u < H(x, y)$ is exactly $\min\{1, H(x, y)\}$.

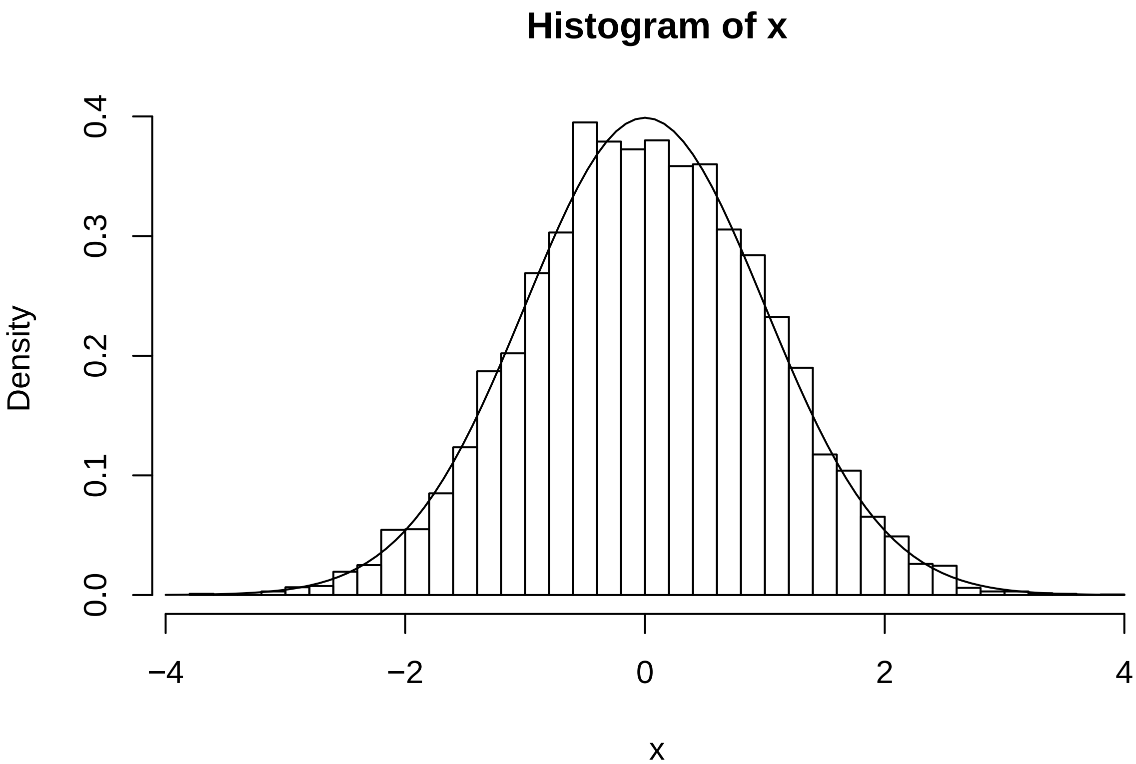
Part 3

Example of **R** implementation:

```
sigma <- 2
n <- 10000 # Chain length
x <- rep(0, n) # Vector to store chain
x[1] <- 0 ## initial value
for(i in 2:n){
  y <- rnorm(1, x[i-1], sigma)
  u <- runif(1)
  H <- dnorm(y)/dnorm(x[i-1])
  if(u<H) x[i]=y else x[i]=x[i-1]
}
plot(x, type="l")
```

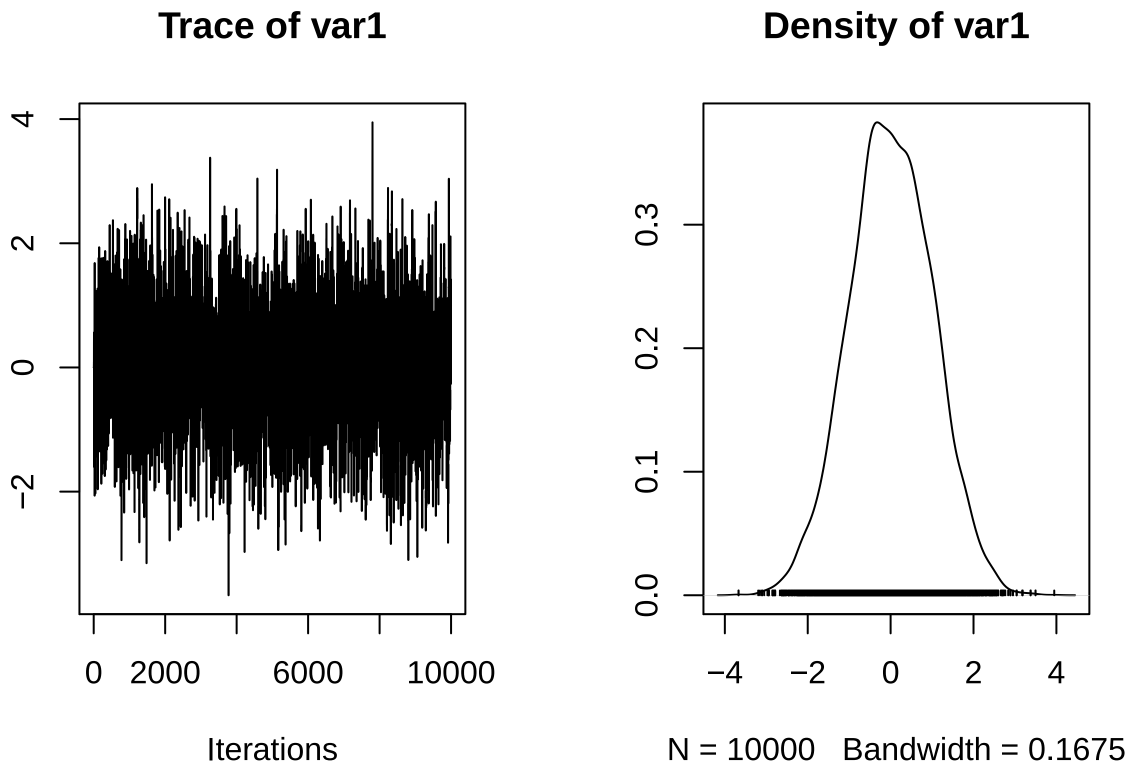


```r
hist(x, prob=TRUE, breaks=50)
curve(dnorm(x), -4, 4, add=TRUE) ## add normal pdf to histogram
```



Plots using `coda` package instead of manual graphics:

```r
library(coda)
x <- mcmc(x) # Convert to coda format
plot(x)
```



Part 4

The estimate is obtained by

```r
mean(x<=-1)
```

```
## [1] 0.1539
```

and the result is close to

```r
pnorm(-1)
```

```
## [1] 0.1586553
```

Part 5

The proposal is independent of the current state x. Furthermore, the proposal follows the same distribution as the target density $\pi(x)$. The acceptance probability is in this case $a(x, y) = 1$.