## Mark and recapture likelihood

Here is an illustration of how well the postulated binomial distributions approximate the the actual relative frequency of counts of the number of recaptured marked animals $(R)$ and the number of unmarked captures (Z).

## Simulating the experiment

```
set.seed(42)
N <- 1000
M <- 100
theta <- 0.05
```

Assume the true unknown population size is $\mathrm{N}=1000$ and we have marked $\mathrm{M}=100$ at a previous visit. Assume the catch probability is $\theta=5 \%$ (important that it is low i.e. $\theta \ll 1$ ). I.e. when we return to the population we catch a random number of animals $K$ which must be approximately binomial with parameters $N$ and $\theta$ (ignoring that we catch without replacement).

A single experiment could be performed in R like this:

```
pop <- factor(c(rep(0, N-M), rep(1,M))) # Population: O unmarked, 1 marked
K <- rbinom(1, size = N, prob = theta) # Random number of catches
s <- sample(pop, size = K) # Single sample from population
tab <- table(s) # Table of counts of "O" and "1"
tab
## s
## 0}
## 50 11
```

We can then repeat this experiment many (nrep) times. In $R$ the replicate() function can be used to run the same code many times:

```
nrep <- }1000
tab <- replicate(nrep, {
    K <- rbinom(1, size = N, prob = theta) # Random number of catches
    s <- sample(pop, size = K) # Single sample from population
    table(s) # Table of counts of "O" and "1"
})
```

The first 10 results for unmarked/marked:

```
tab[,1:10]
\begin{tabular}{lrrrrrrrrrrr} 
\#\# & & & & & & & & \\
\#\# s s & {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} & {\([, 4]\)} & {\([, 5]\)} & {\([, 6]\)} & {\([, 7]\)} & {\([, 8]\)} & {\([, 9]\)} & {\([, 10]\)} \\
\#\# & 0 & 46 & 55 & 32 & 37 & 40 & 46 & 58 & 54 & 52 & 38 \\
\#\# & 1 & 6 & 5 & 3 & 5 & 3 & 10 & 7 & 4 & 5 & 5
\end{tabular}
```

Relative frequency of the number of recaptures compared with a binomial density with parameters $M$ and $\theta$ (observed relative frequency as a black bar, approximate binomial probability as red circle):

```
Rtable <- table(Z=tab[2,])
R <- as.numeric(names(Rtable))
Rfreq <- as.numeric(Rtable)
plot(R, Rfreq/nrep, type = "h")
points(R, dbinom(R, size = M, prob = theta), col = 2)
```



Relative frequency of the number of unmarked captures compared with a binomial density with parameters $U=N-M$ and $\theta$ :

```
Ztable <- table(Z=tab[1,]) # Count
Z <- as.numeric(names(Ztable))
Zfreq <- as.numeric(Ztable)
plot(Z, Zfreq/nrep, type = "h")
points(Z, dbinom(Z, size = N-M, prob = theta), col = 2)
```



