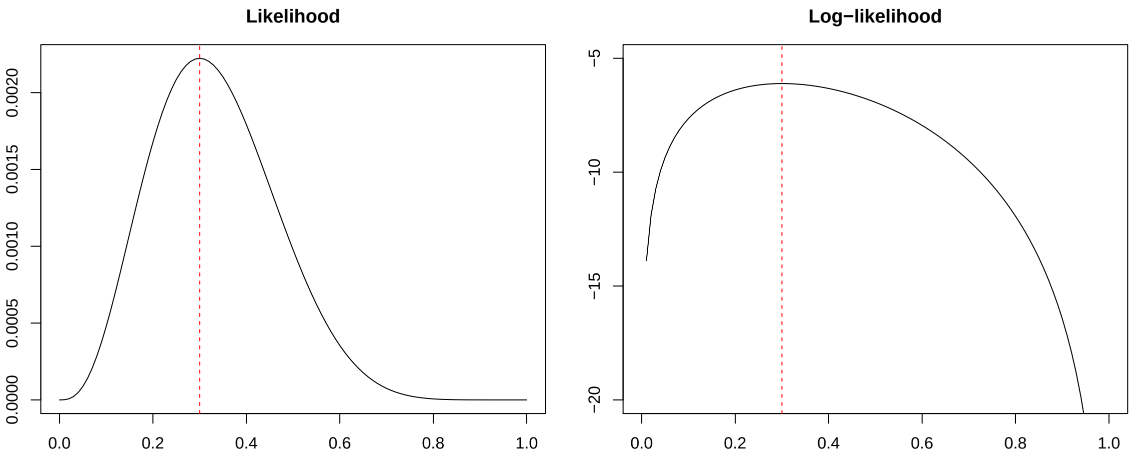
Module 3: Exercises for binomial model

Exercise 1 (solve by inserting code in the Rmd file)

The following figure shows the likelihood and log-likelihood for the binomial model when $n = 10$ and $y = 3$:

```
lik <- function(parm, y, n){parm^y * (1 - parm)^(n - y)}
loglik <- function(parm, y, n){y * log(parm) + (n - y) * log(1 - parm)}
par(mfrow=c(1, 2), mar = c(3,3,3,1))
n <- 10; y <- 3
curve(lik(x, y, n), main = "Likelihood")
abline(v = y/n, lty = 2, col = 2)
curve(loglik(x, y, n), main = "Log-likelihood", ylim = c(-20, -5))
abline(v = y/n, lty = 2, col = 2)
```



Redo the likelihood plot above for $n = 20, y = 6$, for $n = 50, y = 15$ and for $n = 500, y = 150$.

- Do the plots surprise you?
- What do you conclude?

Exercise 2 (solve by hand with pen and paper)

For the binomial model

- Differentiate $l(\theta)$ to obtain $l'(\theta)$ and verify that the solution to $l'(\theta) = 0$ is $\hat{\theta} = y/n$.

Only do the next two bullets if you have solved all other exercises (also Exercise 3):

- Differentiate $l'(\theta)$ to obtain $l''(\theta)$.
- For the MLE it generally holds that the variance of $\hat{\theta}$ is approximately

$$\mathrm{Var}(\hat{\theta}) \approx -1/l''(\hat{\theta}).$$

  Verify by a direct computation that this in fact results in the estimated variance found in the text:

$$\hat{\theta}(1 - \hat{\theta})/n = y(n - y)/n^3.$$

Exercise 3 (solve by inserting code in the Rmd file)

For the Bayesian example with discrete prior:

1. Think about the effect data has on the posterior when compared to the prior.
2. Repeat the computations (mean and variance of posterior) and plots but with $n = 100, y = 30$. Do the results surprise you?
3. Repeat the computations and plots for the case where the prior has a uniform distribution (i.e. if all five values have prior probability 0.20), and $n = 10, y = 3$. What is the "relationship" between the posterior and the likelihood in this case?
4. Lastly, repeat the computations and plots for the case where $\pi(0.1) = \pi(0.3) = \pi(0.5) = \pi(0.9) = 0.01$ and $\pi(0.7) = 0.96$ (still $n = 10, y = 3$). Comment on the result.