Mark and recapture likelihood

Here is an illustration of how well the postulated binomial distributions approximate the the actual relative frequency of counts of the number of recaptured marked animals (R) and the number of unmarked captures (Z).

Simulating the experiment

```
set.seed(42)
N <- 1000
M <- 100
theta <- 0.05
```

Assume the true unknown population size is N=1000 and we have marked M=100 at a previous visit. Assume the catch probability is $\theta = 5\%$ (important that it is low i.e. $\theta << 1$). I.e. when we return to the population we catch a random number of animals K which must be approximately binomial with parameters N and θ (ignoring that we catch without replacement).

A single experiment could be performed in R like this:

```
pop <- factor(c(rep(0, N-M), rep(1,M))) # Population: 0 unmarked, 1 marked
K <- rbinom(1, size = N, prob = theta) # Random number of catches
s <- sample(pop, size = K) # Single sample from population
tab <- table(s) # Table of counts of "0" and "1"
tab
```

```
## s
##  0  1
## 50 11
```

We can then repeat this experiment many (`nrep`) times. In R the `replicate()` function can be used to run the same code many times:

```
nrep <- 10000
tab <- replicate(nrep, {
  K <- rbinom(1, size = N, prob = theta) # Random number of catches
  s <- sample(pop, size = K) # Single sample from population
  table(s) # Table of counts of "0" and "1"
})
```

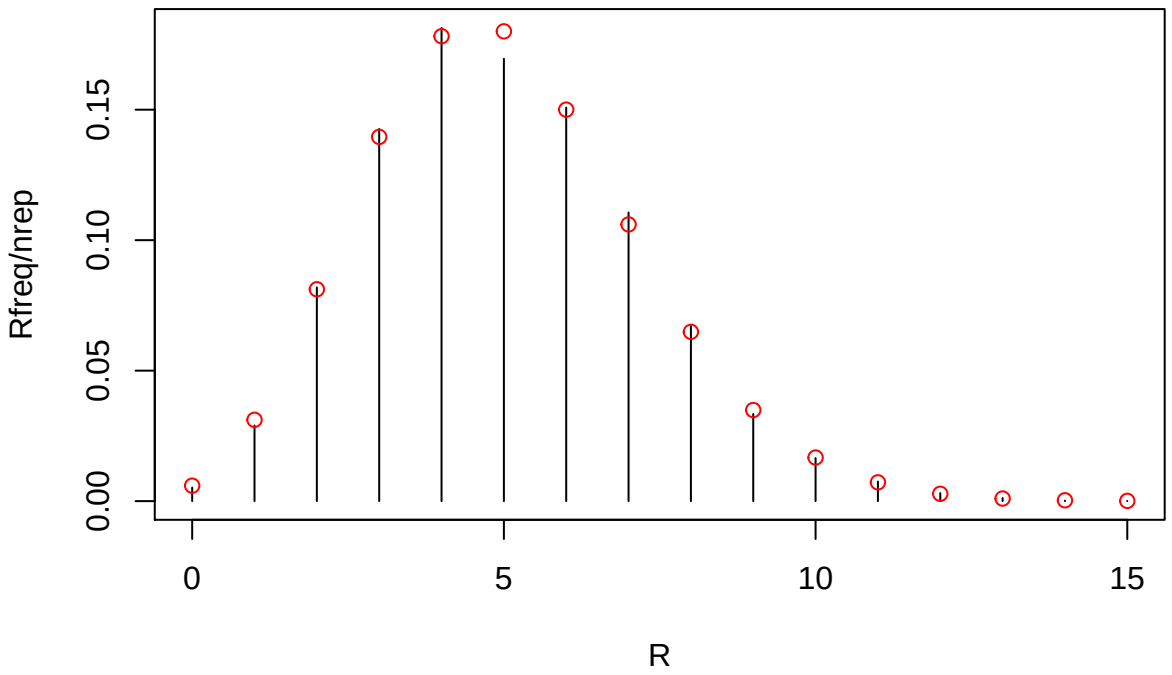
The first 10 results for unmarked/marked:

```
tab[,1:10]
```

```
##
## s   [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##   0   46   55   32   37   40   46   58   54   52    38
##   1    6    5    3    5    3   10    7    4    5     5
```

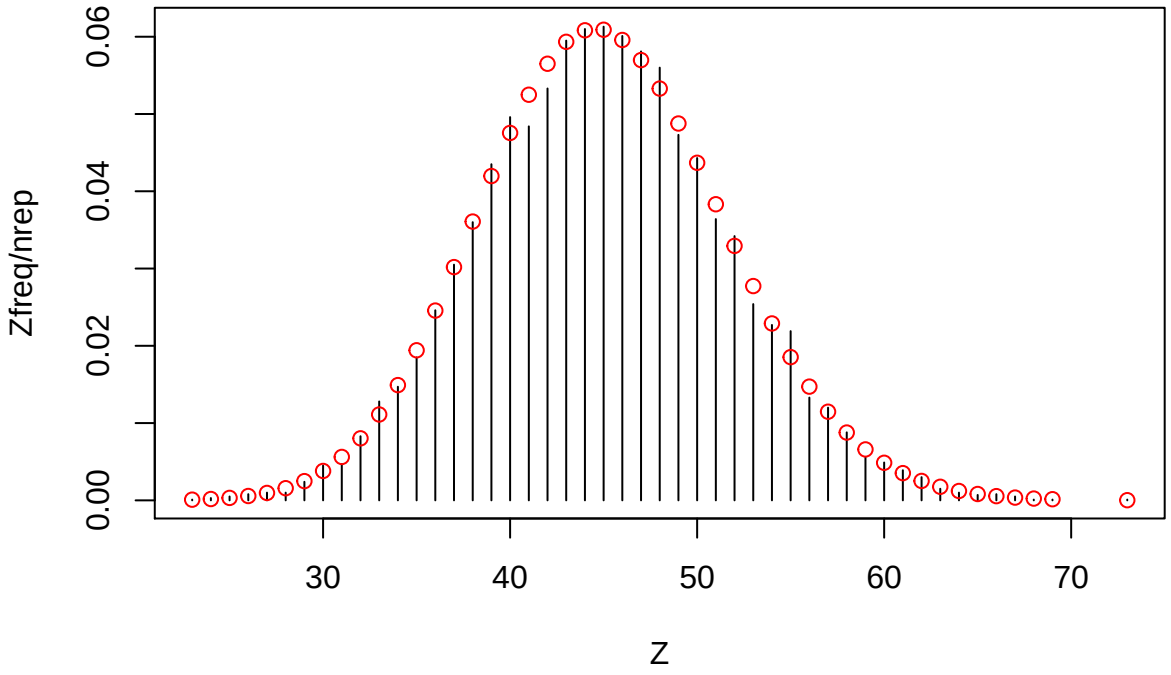
Relative frequency of the number of recaptures compared with a binomial density with parameters M and θ (observed relative frequency as a black bar, approximate binomial probability as red circle):

```
Rtable <- table(Z=tab[2,])
R <- as.numeric(names(Rtable))
Rfreq <- as.numeric(Rtable)
plot(R, Rfreq/nrep, type = "h")
points(R, dbinom(R, size = M, prob = theta), col = 2)
```



Relative frequency of the number of unmarked captures compared with a binomial density with parameters $U = N - M$ and θ:

```
Ztable <- table(Z=tab[1,]) # Count
Z <- as.numeric(names(Ztable))
Zfreq <- as.numeric(Ztable)
plot(Z, Zfreq/nrep, type = "h")
points(Z, dbinom(Z, size = N-M, prob = theta), col = 2)
```


Zfreq/nrep
0.00
0.02
0.04
0.06
30
40
50
60
70
Z