

Exercises for module 6

The Gibbs sampler

1 IQ test

Suppose that a n people have taken an IQ-test. The score x_i obtained by the i th person is assumed to be normally distributed with known precision τ and a mean μ_i which corresponds to the true IQ for that person, ie. $x_i \sim N(\mu_i, \tau)$. Assume that the people taking the test come from a population where the true IQ can be assumed to be normally distributed, ie. $\mu_i \sim N(\mu_G, \tau_G)$. Regarding priors we assume a priori that μ_G and τ_G are independent, and $\mu_G \sim N(\mu_0, \tau_0)$ and $\tau_G \sim \text{Gamma}(\alpha, \beta)$.

1. Determine the joint distribution $\pi(x_1, \dots, x_n, \mu_1, \dots, \mu_n, \mu_G, \tau_G)$.
2. Determine the full conditionals, $\pi(\mu_1 | \mu_2, \dots, \mu_n, \mu_G, \tau_G, x_1, \dots, x_n)$ etc.
Hint: Have a look at the results for the case of n independent samples x_1, \dots, x_n from the same normal distributon $N(\mu, \tau)$.
3. Specify a Gibbs sampler for sampling $\pi(\mu_1, \dots, \mu_n, \mu_G, \tau_G | x_1, \dots, x_n)$.

2 Radiocarbon dating

Following Lee (2003, p. 263) consider the following example for archeology: Assume that for each of three samples we have measured the date as x_1, x_2 and x_3 and a reasonable approximation is $x_i \sim N(\mu_i, \tau_i)$, where μ_i is the true age and τ_i is known. It is further known that the age of the samples are positive and below some upper limit k . In addition the time order of the three samples is known, that is $\mu_1 < \mu_2 < \mu_3$. Thus, as a joint prior on (μ_1, μ_2, μ_3) we propose

$$\pi(\mu_1, \mu_2, \mu_3) \propto \begin{cases} c & \text{if } 0 < \mu_1 < \mu_2 < \mu_3 < k \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where c is a positive constant.

1. Determine the joint posterior pdf for the three mean values, ie. $\pi(\mu_1, \mu_2, \mu_3 | x_1, x_2, x_3)$.
2. Determine the full conditionals, ie. $\pi(\mu_1 | \mu_2, \mu_3, x_1, x_2, x_3)$ etc.
Notice: These distributions are non-standard.
3. Specify a Gibbs sampler for sampling of the posterior. How would you generate samples from the non-standard distributions above?