Exercises for module 1
Basics of probability theory

Exercise 1

A fair coin is tossed n times (where n is a given positive integer).

1. Specify the state space Ω and the probability measure P for all possible realisations.

2. Let A be the event "the coin shows up with both a head and a tail" and let B be the event that "there is at most one tail". Determine $P(A)$, $P(B)$, and $P(A \cap B)$.

3. Are A and B independent events?

Exercise 2

A number X is picked uniformly at random on the interval $[0, 1]$, that is, for any $I \subseteq [0, 1]$, we have $P(X \in I) = $ length of I. We say that X is *uniformly distributed between 0 and 1* and write $X \sim \mathrm{unif}(0, 1)$.

1. Specify the distribution function, density function, mean, and variance of X.

2. What is the probability that the first decimal of X is equal to 1.

Exercise 3

A random variable X is said to follow an *exponential distribution with parameter $\lambda > 0$* if X has density

$$f_X(x) = \lambda \exp(-\lambda x), \qquad x > 0$$

(meaning that $f_X(x) = 0$ if $x \leq 0$).

1. Determine the distribution function and the mean of X.

2. For any numbers $s > 0$ and $t > 0$, find $P(X > t + s | X > s)$ and interpret the result.