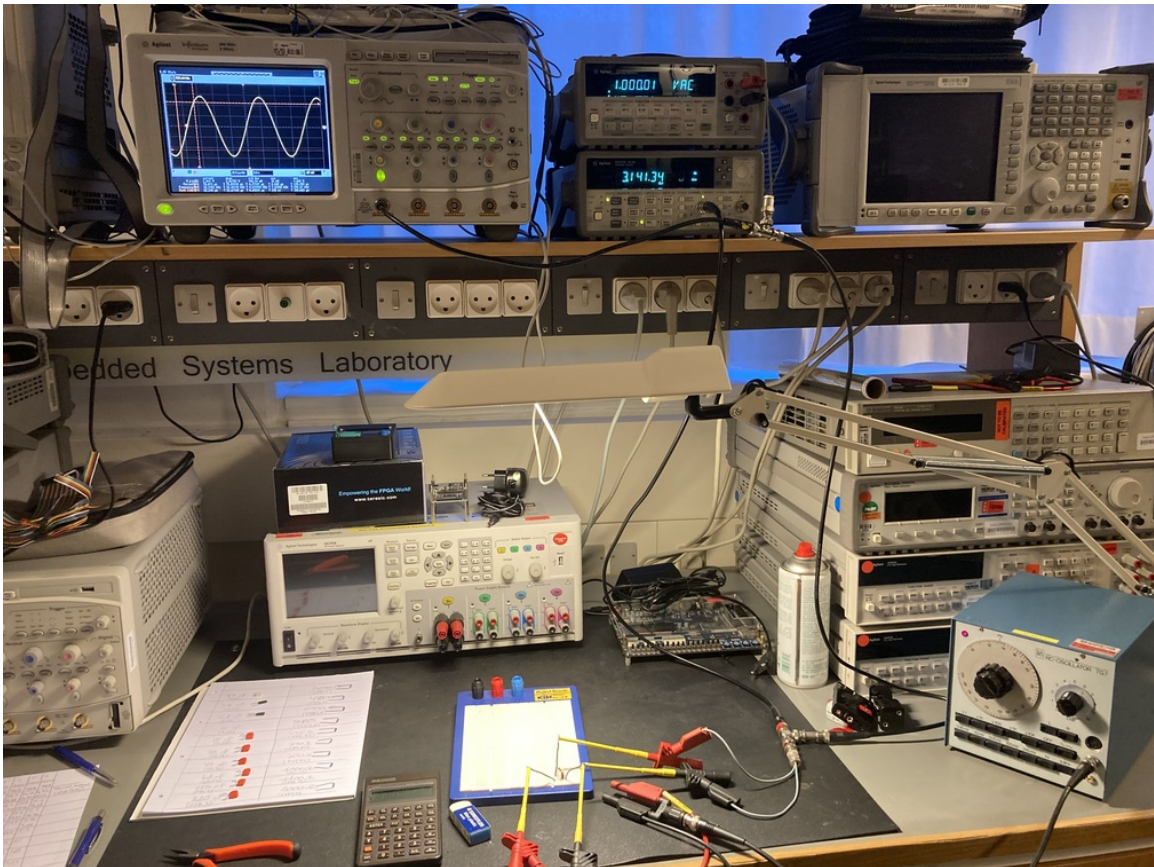
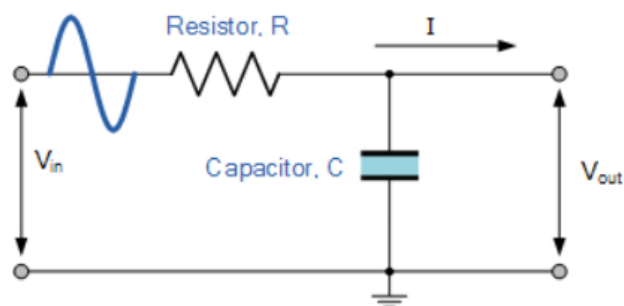


Exam exercise for Module 4: Low pass filter



Setup in lab.

First order low pass filter



We shall study data associated with the displayed circuit. The fundamental characteristics associated with the circuit is:

- R is the resistance measured in Ohm
- C is the capacitance measured in Farad
- f_c is the 3dB cut-off frequency measured in Hertz

In theory the relation between these quantities is

$$f_c = \frac{1}{2\pi RC}$$

Data

Peter Koch has spent quite some time in the lab producing data. Peter has measured R , C and f_c for different combinations of nominal R and C values.

Four resistors were used with nominal R values:

- 1, 1.1, 4.75 and 5.9 kOhm

Seven capacitors were used with nominal C values:

- 47, 56, 68, 330, 470, 560 and 680 nF

For each component, the actual R or C value has been measured once. Moreover, for each combination of capacitor and resistor, the frequency has been measured, that is, in total $4 \times 7 = 28$ measurements of frequency were made.

First load the data:

```
load(url("https://asta.math.aau.dk/datasets?file=RC_data.RData"))
head(RC_data)
```

```
##      R_nom   C_nom R_measured C_measured f_c_measured
## 4      1000 4.7e-09      997.49   4.62e-09         29080
## 5      1100 4.7e-09     1100.46   4.62e-09         26500
## 8      4750 4.7e-09     4724.70   4.62e-09          6380
## 9      5900 4.7e-09     5880.50   4.62e-09          5120
## 14     1000 5.6e-09      997.49   5.25e-09         25700
## 15     1100 5.6e-09     1100.46   5.25e-09         23440
```

On a logarithmic scale the model is:

- $\ln(f_c\text{true}) = -\ln(2\pi) - \ln(R\text{true}) - \ln(C\text{true})$

where “true” refers to the exact value of the variable for the components tested.

What we are actually measuring is

- $\ln(f_c\text{measured}) = \ln(f_c\text{true}) + f_c\text{error}$
- $\ln(R\text{measured}) = \ln(R\text{true}) + R\text{error}$
- $\ln(C\text{measured}) = \ln(C\text{true}) + C\text{error}$

1) Show that:

- $\log(f_c\text{measured}) = \log_{fc}\text{predict} + R\text{error} + C\text{error} + f_c\text{error}$

where

- $\log_{fc}\text{predict} = -\log(2\pi) - \log(R\text{measured}) - \log(C\text{measured})$

2) Argue that if the meters have no systematic errors, then $(\log_{fc}\text{predict}, \log(f_c\text{measured}))$ should vary around the identity line.

- Calculate and plot these points. You can use the code below to add the identity line to the plot.

```
# gf_point(...) %>% #plot the points
# gf_abline(slope=~1, intercept=~0) #adds the identity line
```

- Argue that the plot calls for a linear calibration of $\log(f_c\text{measured})$.

3) Fit a simple linear regression of $\log(f_c\text{measured})$ on $\log_{fc}\text{predict}$. Argue that

- the intercept is significantly different from zero.
- the slope is significantly different from one.

This shows that the meters must have systematic errors.

4) In the light of your conclusions do a calibration of $\log(\text{fc_measured})$ and call it $\log_{\text{fc_corrected}}$.

- Make a plot of $(\log_{\text{fc_predict}}, \log_{\text{fc_corrected}})$

5) The data version of the RC-model is now

- $\log_{\text{fc_corrected}} = -\log(2\pi) - \log(R_{\text{measured}}) - \log(C_{\text{measured}}) + \text{error}$

Fit a multiple regression of $\log_{\text{fc_corrected}}$ on $\log(R_{\text{measured}})$ and $\log(C_{\text{measured}})$ and test the hypotheses

- intercept is equal to $-\log(2\pi)$.
- slope of $\log(R_{\text{measured}})$ is equal to -1.
- slope of $\log(C_{\text{measured}})$ is equal to -1.

6) If the fitted model is called `fit`, then the residuals is obtained by `resid(fit)`. Investigate whether the residuals follow a normal distribution using

- a `qqnorm` plot of the residuals
- Geary's test
- Godness-of-fit test with 5 bins
- Shapiro-Wilks test

What is your conclusion?