Quality Control

The ASTA team

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1 Quality control

1.1 Quality control chart

Control charts are used to routinely monitor quality.

Two major types:

- **univariate control**: a graphical display (chart) of one quality characteristic
- **multivariate control**: a graphical display of a statistic that summarizes or represents more than one quality characteristic

The control chart shows

- the value of the quality characteristic versus the sample number or versus time
- a **center line** (CL) that represents the mean value for the in-control process
- an **upper control limit** (UCL) and a **lower control limit** (LCL)

The control limits are chosen so that almost all of the data points will fall within these limits **as long as the process remains in-control**.

1.2 Example

```
library(qcc)
data(pistonrings)
head(pistonrings,3)
```

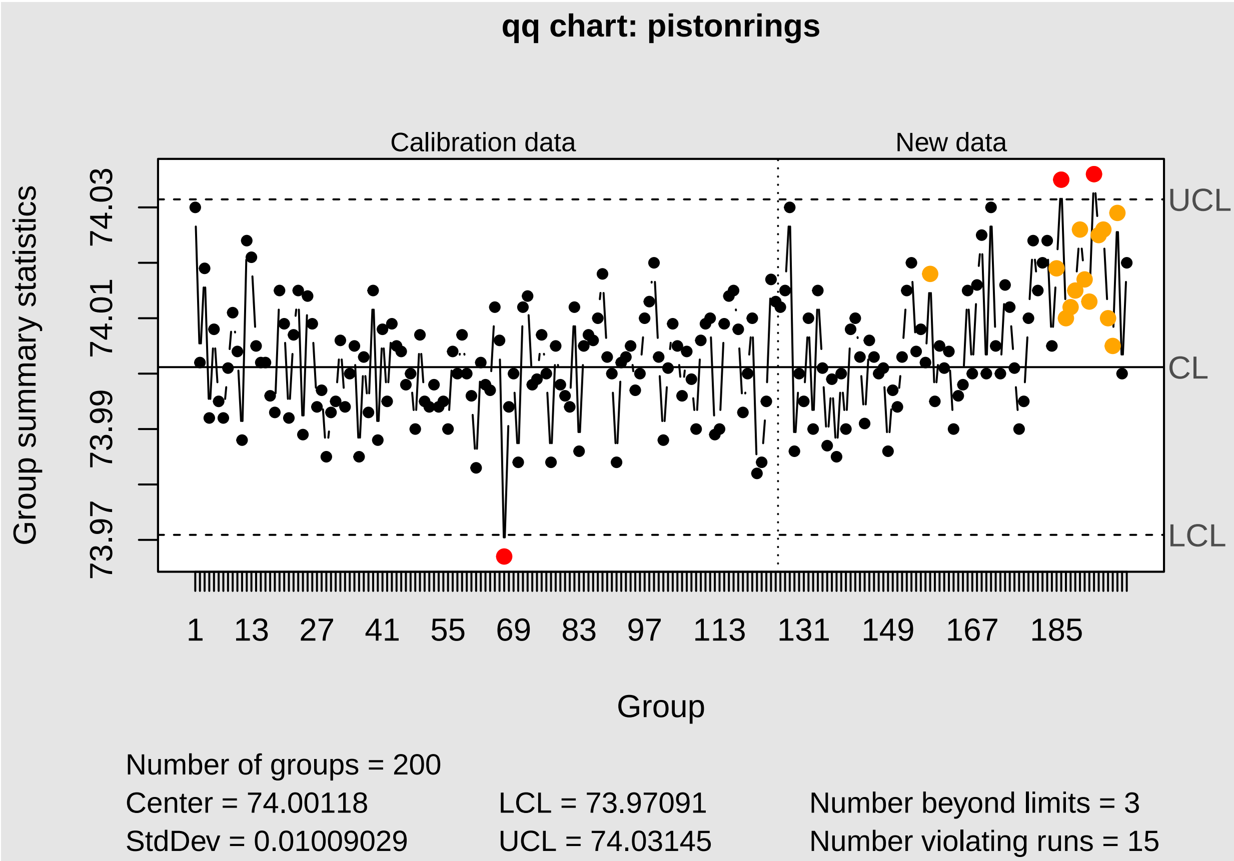
```
##   diameter sample trial
## 1   74.030      1  TRUE
## 2   74.002      1  TRUE
## 3   74.019      1  TRUE
```

Piston rings for an automotive engine are produced by a forging process. The inside diameter of the rings manufactured by the process is measured on 25 samples(`sample=1,2,..,25`), each of size 5, for the control phase I (`trial=TRUE`), when preliminary samples from a process being considered 'in-control' are used to construct control charts. Then, further 15 samples, again each of size 5, are obtained for phase II (`trial=FALSE`).

Reference:

Montgomery, D.C. (1991) Introduction to Statistical Quality Control, 2nd ed, New York, John Wiley & Sons, pp. 206-213

1.3 Example



We shall treat different methods for determining LCL,CL and UCL. In that respect, it is crucial that we have

- **phase I data**, where the process is in-control.
- These data are used to determine LCL,CL and UCL.

1.4 The simple six sigma model

Assume that measurements

- is a sample, i.e they are independent
- they have a normal distribution
- we know the mean μ_0 and standard deviation σ_0.

  In this case we dont need phase I data.
- CL=μ_0.
- LCL=$\mu_0 - k\sigma_0$.
- UCL=$\mu_0 + k\sigma_0$.

The only parameter to determine is k.

We dont want to give a lot of false warnings, and a popular choise is

- k=3, known as the 3*sigma rule.
- The probability of a measurement outside the control limits is then 0.27%, when the proces is in-control.

This means that the span of allowable variation is $6\sigma_0$.

The concept "Six Sigma'' has become a mantra in many industrial communities.

1.5 Average Run Length (ARL)

Let `pOut` denote the probability that a measurement is outside the control limits. On average this means that we need 1/`pOut` observations before we get an outlier.

This is known as the *the Average Run Length*:

$$AVL = \frac{1}{\texttt{pOut}}$$

An in-control process with the 3*sigma rule has AVL

```r
round(1/(2*pdist("norm", -3, plot = FALSE)))
```

```
## [1] 370
```

An in-control process with AVL=500 has k*sigma rule, where k equals

```r
-qdist("norm", (1/2)*(1/500), plot = FALSE)
```

```
## [1] 3.090232
```

1.6 Types of quality control charts.

Depending on the type of control variable, there are various types of control charts.

chart	distribution	statistic	example
xbar	normal	mean	means of a continuous process variable
S	normal	standard deviation	standard deviations of a continuous process variable
R	normal	range	ranges of a continuous process variable
p	binomial	proportion	percentage of faulty items
c	poisson	count	number of faulty items during a workday

#Continuous process variable

1.7 Continuous process variable

Phase I data:

- m samples with n measurements in each sample.
- For sample $i = 1, 2, \ldots m$ calculate mean $\bar{x}_i$ and standard deviation s_i.

- Calculate
$$\bar{x} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i \quad \text{and} \quad \bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i$$

  When the sample is normal, it can be shown that $\bar{s}$ is a biased estimate of the true standard deviation σ:
- $E(\bar{s}) = c_4(n)\sigma$
- $c_4(n)$ is tabulated in textbooks and available in the `qcc` package.

  Unbiased estimate of σ:
$$\hat{\sigma}_1 = \frac{\bar{s}}{c_4(n)}$$

  Furthermore $\bar{s}$ has estimated standard error
$$se(\bar{s}) = \bar{s}\frac{\sqrt{1 - c_4(n)^2}}{c_4(n)}$$

1.8 xbar chart

$$\begin{aligned} \text{UCL:} &\quad \bar{x} + 3\frac{\hat{\sigma}_1}{\sqrt{n}} \\ \text{CL:} &\quad \bar{x} \\ \text{LCL:} &\quad \bar{x} - 3\frac{\hat{\sigma}_1}{\sqrt{n}} \end{aligned}$$

This corresponds to

- The probability of a measurement outside the control limits is 0.27%.

  If we want to change this probability, we need another z-score. E.g if we want to lower this probability to 0.1%, then 3 should be substituted by 3.29.

1.9 Example

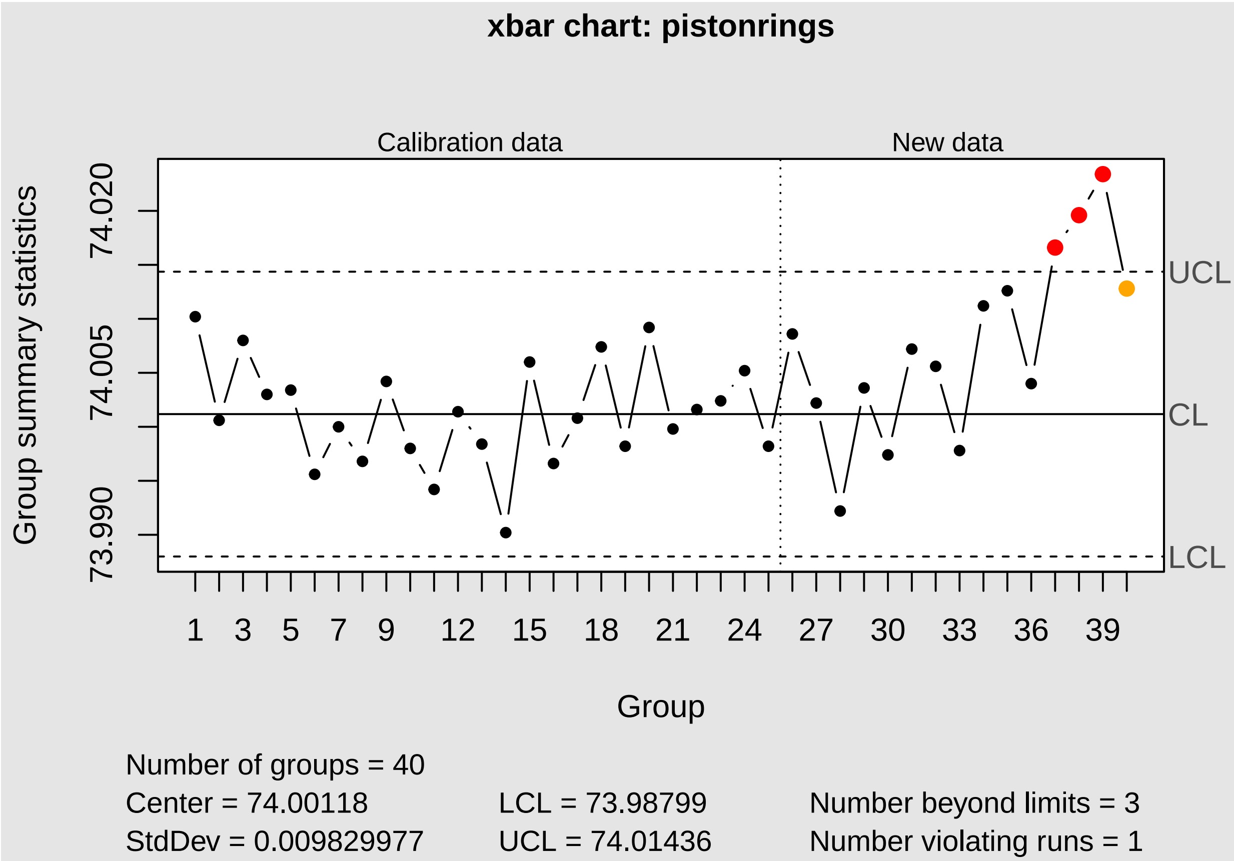
```
phaseI  <- matrix(pistonrings$diameter[1:125]   , nrow=25, byrow=TRUE)
phaseII <- matrix(pistonrings$diameter[126:200], nrow=25, byrow=TRUE)
h <- qcc(phaseI, type = "xbar", std.dev = "UWAVE-SD",
         newdata = phaseII, title = "xbar chart: pistonrings")
```

- `phaseI` is a matrix with $m = 25$ rows, where each row is a sample of size $n = 5$.
- Similarly `phaseII` has 15 samples.

The function `qcc` calculates the necessary statistics and optionally makes a plot.

- `phaseI` and `type=` are the only arguments required.
- We want that the limits are based on the unweighted average of standard deviations - `UWAVE-SD`. This is not the default.
- We also want to evaluate the phase II data: `newdata=phaseII`.
- Optionally, we can specify the title on the plot.

1.10 Example



Besides limits we are also told whether the process is above/below CL for 7 or more consecutive samples (yellow dots).

`run.length=7` is default, but may be changed. If we e.g. want this to happen with probability 0.2%, then we specify `run.length=10`.

1.11 S chart: Monitoring variability

In most situations, it is crucial to monitor the process mean.

But it may also be a problem if the variability in "quality" gets too high.

In that respect, it is relevant to monitor the standard deviation, which is done by the S-chart:

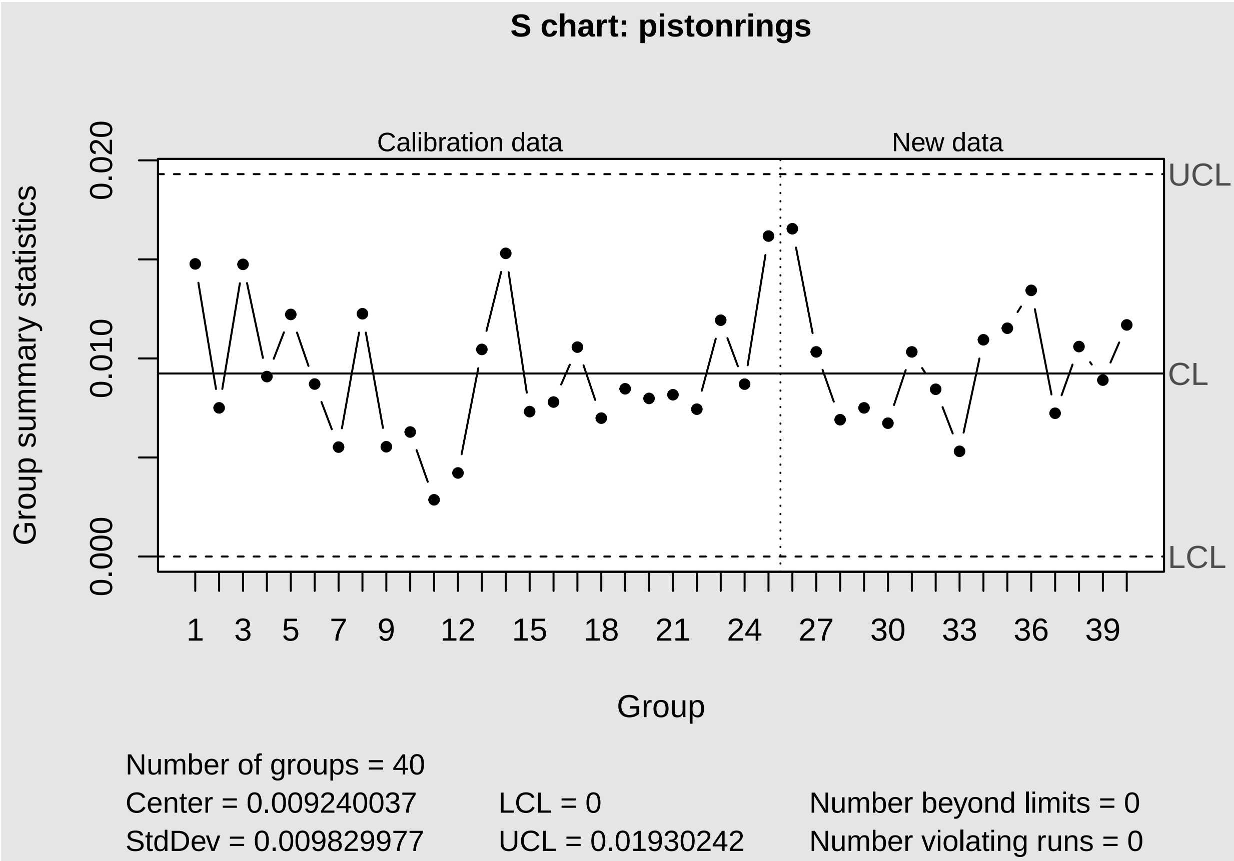
$$\begin{aligned} \text{UCL:} &\quad \bar{s} + 3se(\bar{s}) \\ \text{CL:} &\quad \bar{s} \\ \text{LCL:} &\quad \bar{s} - 3se(\bar{s}) \end{aligned}$$

$$se(\bar{s}) = \bar{s}\frac{\sqrt{1 - c_4(n)^2}}{c_4(n)}$$

Where 3 may be substituted by some other z-score depending on the required confidence level.

```
h <- qcc(phaseI,type="S", newdata=phaseII, title="S chart: pistonrings")
```

1.12 S chart example



Remark that the plot does not allow values below zero.

Quite sensible when we are talking about standard deviations.

1.13 R chart: Range statistics

If the sample size is relatively small ($n \leq 10$), it is custom to use the range R instead of the standard deviation. The range of a sample is simply the difference between the largest and smallest observation.

When the sample is normal, it can be shown that:

- $E(\bar{R}) = d_2(n)\sigma$, where $\bar{R}$ is the average of the m sample ranges.
- $d_2(n)$ is tabulated in textbooks and available in the `qcc` package.

  Unbiased estimate of σ:

  $$\hat{\sigma}_2 = \frac{\bar{R}}{d_2(n)}$$

  Furthermore $\bar{R}$ has estimated standard error

  $$se(\bar{R}) = \bar{R}\frac{d_3(n)}{d_2(n)}$$

  $d_3(n)$ is tabulated in textbooks and available in the `qcc` package.

1.14 Charts based on R

xbar chart based on $\bar{R}$:

$$\begin{aligned} \text{UCL:} \quad & \bar{x} + 3\frac{\hat{\sigma}_2}{\sqrt{n}} \\ \text{CL:} \quad & \bar{x} \\ \text{LCL:} \quad & \bar{x} - 3\frac{\hat{\sigma}_2}{\sqrt{n}} \end{aligned}$$

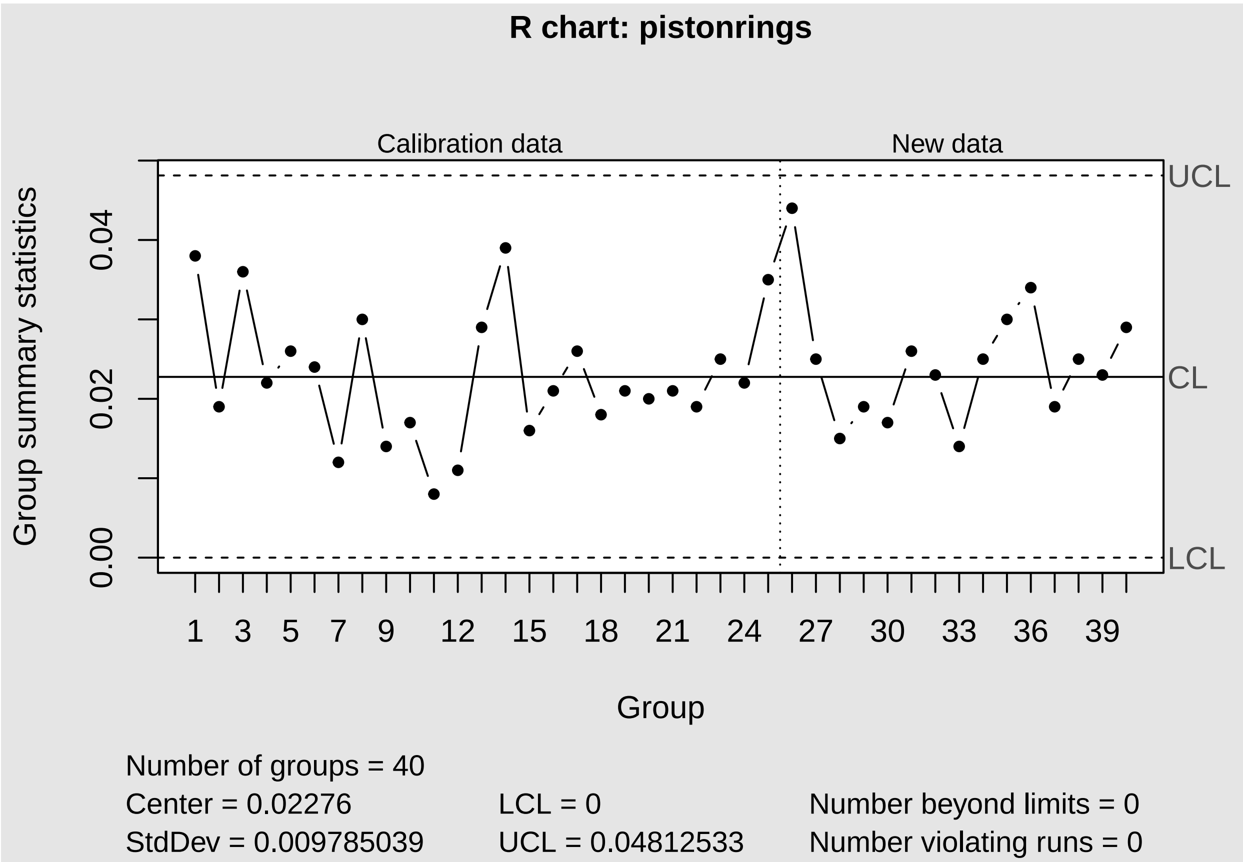
This is actually the default in the `qcc` package.

R chart to monitor variability:

$$\begin{aligned} \text{UCL:} \quad & \bar{R} + 3se(\bar{R}) \\ \text{CL:} \quad & \bar{R} \\ \text{LCL:} \quad & \bar{R} - 3se(\bar{R}) \end{aligned}$$

1.15 R chart example

```
h <- qcc(phaseI, type="R", newdata=phaseII, title="R chart: pistonrings")
```



2 Binomial process variable

2.1 Binomial variation

Let us suppose that the production process operates in a stable manner such that

- the probability that an item is defect is p.

- successive items produced are independent

  In a random sample of n items, the number D of defective items follows a binomial distribution with parameters n and p.

  Unbiased estimate of p:

  $$\hat{p} = \frac{D}{n}$$

  which has standard error

  $$se(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

2.2 p chart

Data from phase I:

- m samples with estimated proportions $\hat{p}_i, \quad i = 1, \ldots, m$
- $\bar{p}$ is the average of the estimated proportions.

p chart:

$$\begin{aligned} \text{UCL:} \quad & \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\ \text{CL:} \quad & \bar{p} \\ \text{LCL:} \quad & \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \end{aligned}$$

2.3 Example

```r
data(orangejuice)
head(orangejuice, 3)
```

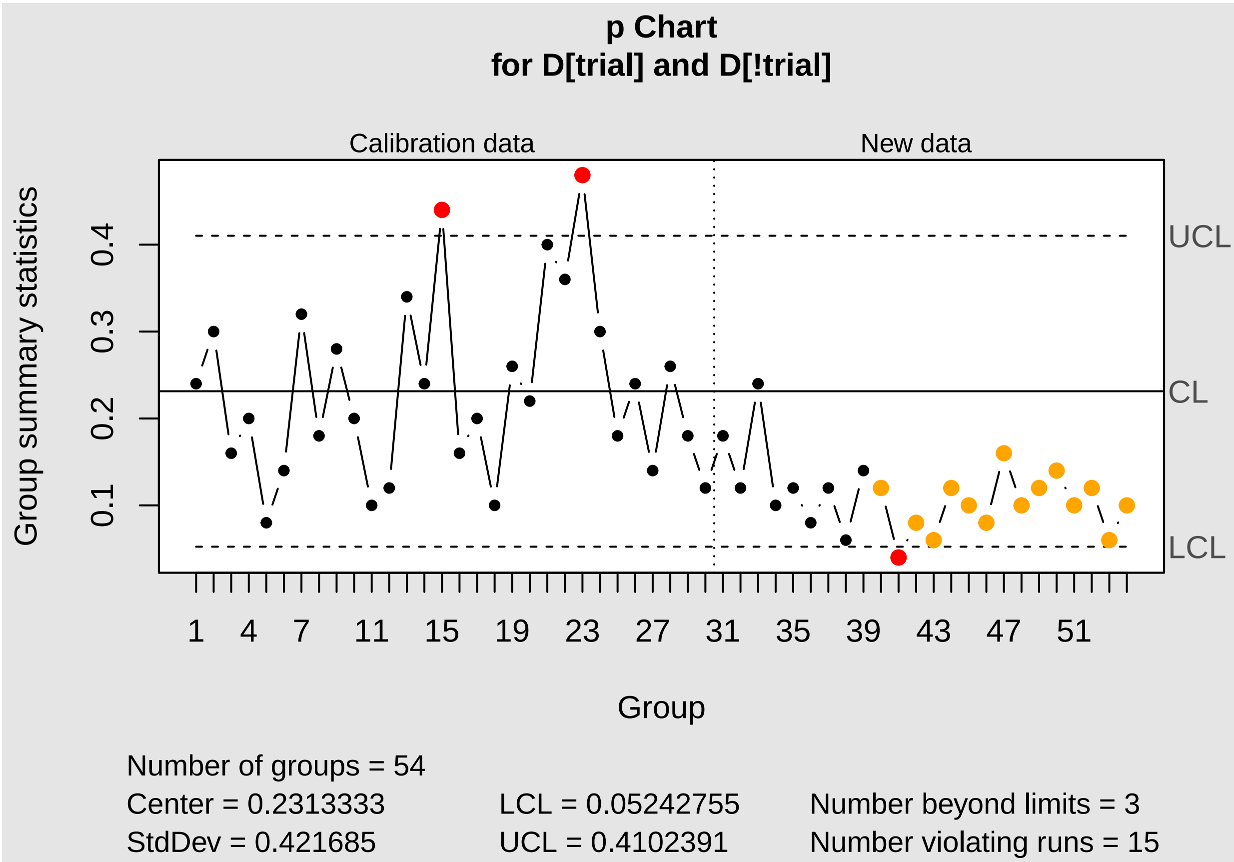
```
##   sample  D size trial
## 1      1 12   50  TRUE
## 2      2 15   50  TRUE
## 3      3  8   50  TRUE
```

Production of orange juice cans.

- The data were collected as 30 samples of 50 cans.
- The number of defective cans `D` were observed.
- After the first 30 samples, a machine adjustment was made.
- Then further 24 samples were taken from the process.

```r
with(orangejuice,
     qcc(D[trial], sizes=size[trial], type="p",
         newdata=D[!trial], newsizes=size[!trial]))
```

2.4 Example



The machine adjustment after sample 30 has had an obvious effect.

The chart should be recalibrated.

3 Poisson process variable

3.1 Poisson variation

Let us suppose that the production process operates in a stable manner such that

- defective items are produced at a constant rate

  The number D of defective items over a time interval of some fixed length follows a poisson distribution with mean value c.

  Unbiased estimate of c:

  $$\hat{c} = D$$

  which has standard error

  $$se(\hat{c}) = \sqrt{c}$$

3.2 c chart

Data from phase I:

- m sampling periods with mean estimates $\hat{c}_i, \quad i = 1, \dots, m$
- $\bar{c}$ is the average of the estimated means.

c chart:

$$\begin{aligned} \text{UCL:} \quad & \bar{c} + 3\sqrt{\bar{c}} \\ \text{CL:} \quad & \bar{c} \\ \text{LCL:} \quad & \bar{c} - 3\sqrt{\bar{c}} \end{aligned}$$