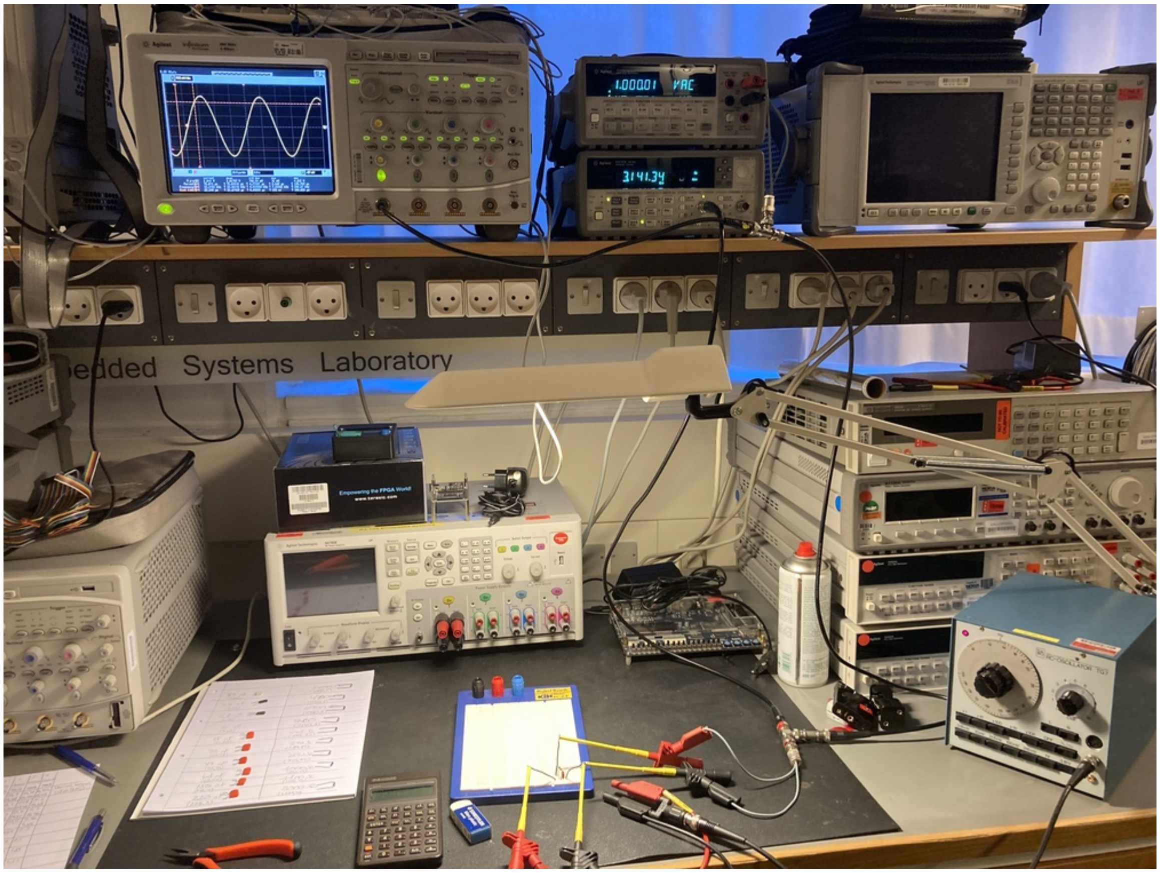
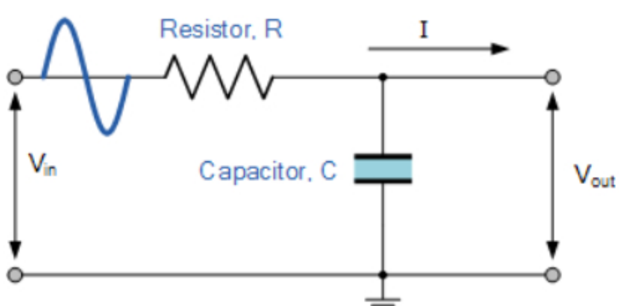
Exam exercise for Module 4: Low pass filter



Setup in lab.

First order low pass filter



We shall study data associated with the displayed circuit. The fundamental characteristics associated with the circuit is:

- R is the resistance measured in Ohm
- C is the capacitance measured in Farad
- f_c is the 3dB cut-off frequency measured in Hertz

In theory the relation between these quantities is

$$f_c = \frac{1}{2\pi RC}$$

Data

Peter Koch has spent quite some time in the lab producing data. Peter has measured R, C and f_c for different combinations of nominal R and C values.

Four resistors were used with nominal R values:

- 1, 1.1, 4.75 and 5.9 kOhm

Seven capacitors were used with nominal C values:

- 47, 56, 68, 330, 470, 560 and 680 nF

For each component, the actual R or C value has been measured once. Moreover, for each combination of capacitor and resistor, the frequency has been measured, that is, in total $4x7 = 28$ measurements of frequency were made.

First load the data:

```
load(url("https://asta.math.aau.dk/datasets?file=RC_data.RData"))
head(RC_data)
```

```
##    R_nom   C_nom R_measured C_measured f_c_measured
## 4   1000 4.7e-09     997.49   4.62e-09        29080
## 5   1100 4.7e-09    1100.46   4.62e-09        26500
## 8   4750 4.7e-09    4724.70   4.62e-09         6380
## 9   5900 4.7e-09    5880.50   4.62e-09         5120
## 14  1000 5.6e-09     997.49   5.25e-09        25700
## 15  1100 5.6e-09    1100.46   5.25e-09        23440
```

On a logarithmic scale the model is:

- `ln(fc_true)=-ln(2*pi)-ln(R_true)-ln(C_true)`

where "true" refers to the exact value of the variable for the components tested.

What we are actually measuring is

- `ln(fc_measured)=ln(fc_true)+fc_error`
- `ln(R_measured)=ln(R_true)+R_error`
- `ln(C_measured)=ln(C_true)+C_error`

1) Show that:

- `log(fc_measured)=log_fc_predict+R_error+C_error+fc_error`

where

- `log_fc_predict= -log(2*pi)-log(R_measured)-log(C_measured)`

2) Argue that if the meters have no systematic errors, then `(log_fc_predict,log(fc_measured))` should vary around the identity line.

- Calculate and plot these points. You can use the code below to add the identity line to the plot.

```
# gf_point(...) %>% #plot the points
# gf_abline(slope=~1,intercept=~0) #adds the identity line
```

- Argue that the plot calls for a linear calibration of `log(fc_measured)`.

3) Fit a simple linear regression of `log(fc_measured)` on `log_fc_predict`. Argue that

- the intercept is significantly different from zero.
- the slope is significantly different from one.

This shows that the meters must have systematic errors.

4) In the light of your conclusions do a calibration of `log(fc_measured)` and call it `log_fc_corrected`.

- Make a plot of `(log_fc_predict,log_fc_corrected)`

5) The data version of the RC-model is now

- `log_fc_corrected=-log(2*pi)-log(R_measured)-log(C_measured)+error`

Fit a multiple regression of `log_fc_corrected` on `log(R_measured)` and `log(C_measured)` and test the hypotheses

- intercept is equal to `-log(2*pi)`.
- slope of `log(R_measured)` is equal to -1.
- slope of `log(C_measured)` is equal to -1.

6) If the fitted model is called `fit`, then the residuals is obtained by `resid(fit)`. Investigate whether the residuals follow a normal distribution using

- a `qqnorm` plot of the residuals
- Geary's test
- Godness-of-fit test with 5 bins
- Shapiro-Wilks test

What is your conclusion?