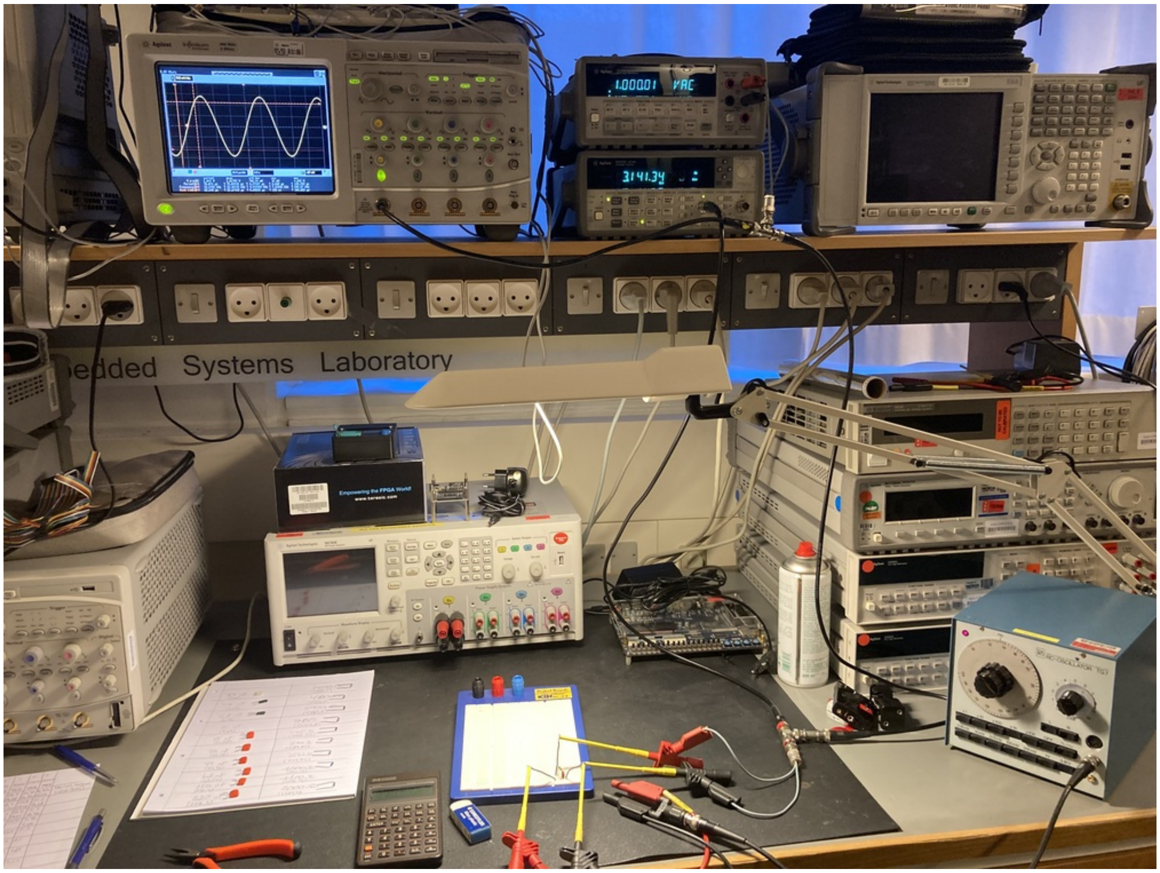
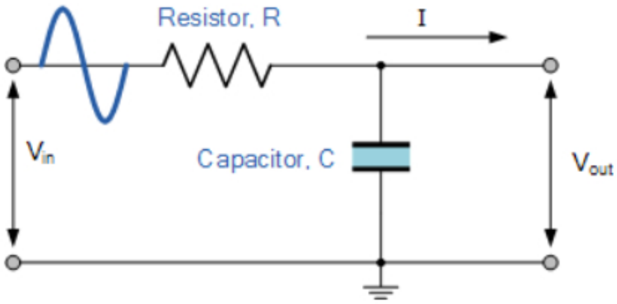
Exam exercise for Module 4: Low pass filter



Setup in lab.

First order low pass filter



We shall study data associated with the displayed circuit.

The fundamental characteristics associated with the circuit is:

- R is the resistance measured in Ohm
- C is the capacitor measured in Farad
- f_c is the 3dB cut-off frequency measured in Hertz

In theory the relation between these quantities is

$$f_c = \frac{1}{2\pi RC}$$

Data

Peter Koch has spent quite some time in the lab producing data.

For different combinations of nominal R and C values Peter has measured R, C and f_c.

Nominal R values:

- 1, 1.1, 4.75 and 5.9 kOhm

Nominal C values:

- 47, 56, 68, 330, 470, 560 and 680 nF

The 3 parameters have been measured for each combination of nominal values , that is, 4 resistor measurements, 7 capacitor measurements and 4x7=28 measurements of frequency.

Load data:

```
load(url("https://asta.math.aau.dk/datasets?file=RC_data.RData"))
head(RC_data)
```

```
##    R_nom   C_nom R_measured C_measured f_c_measured
## 4   1000 4.7e-09     997.49   4.62e-09        29080
## 5   1100 4.7e-09    1100.46   4.62e-09        26500
## 8   4750 4.7e-09    4724.70   4.62e-09         6380
## 9   5900 4.7e-09    5880.50   4.62e-09         5120
## 14  1000 5.6e-09     997.49   5.25e-09        25700
## 15  1100 5.6e-09    1100.46   5.25e-09        23440
```

On a logarithmic scale the model is:

- `log(fc_true)=-log(2*pi)-log(R_true)-log(C_true)`

where true refers to that this is the exact value of the parameter.

Actually we are measuring

- `log(fc_measured)=log(fc_true)+fc_error`
- `log(R_measured)=log(R_true)+R_error`
- `log(C_measured)=log(C_true)+C_error`

1) Show that:

- `log(fc_measured)=log_fc_predict+R_error+C_error+fc_error`
- where `log_fc_predict= -log(2*pi)-log(R_measured)-log(C_measured)`

2) Argue that if the meters have no systematic errors, then

- `(log_fc_predict,log(fc_measured))` should vary around the identity line

Calculate and plot these points.

Argue that the plot calls for a linear calibration of `log(fc_measured)`.

3) Fit a simple linear regression of `log(fc_measured)` on `log_fc_predict`

Argue that

- the intercept is significantly different from zero.
- the slope is significantly different from one.

Meaning that meters must have systematic errors.

4) In the light of your conclusions do a calibration of `log(fc_measured)` and call it `log_fc_corrected`.

Do a plot of `(log_fc_predict,log_fc_corrected)`

5) The data version of the RC-model is now

- `log_fc_corrected=-log(2*pi)-log(R_measured)-log(C_measured)+error`

Fit a multiple regression of `log_fc_corrected` on `log(R_measured)`and `log(C_measured)` and test the hypotheses

- intercept is equal to `-log(2*pi)`
- slope of `log(R_measured)` is equal to -1.
- slope of `log(C_measured)` is equal to -1.

6) If the fitted model is called `fit`, then the residuals is obtained by `resid(fit)`.

Do a `qqnorm` plot of the residuals and a Shapiro-Wilks test of normality of the residuals.

What is your conclusion?