Moving average model of order 1

Simulation of MA(1)

- The series w_t is white noise with zero mean and variance σ_w^2. For the following moving average models, find the autocorrelation function. In addition, simulate 100 observations from each model in R, compare the time plots of the simulated series, and comment on how the two series might be distinguished.
- $$x_t = w_t + \frac{1}{2} w_{t-1}$$
- $$x_t = w_t + 2w_{t-1}$$

Theoretical ACF

- For model 1 with $\beta_1 = 0.5$ we have

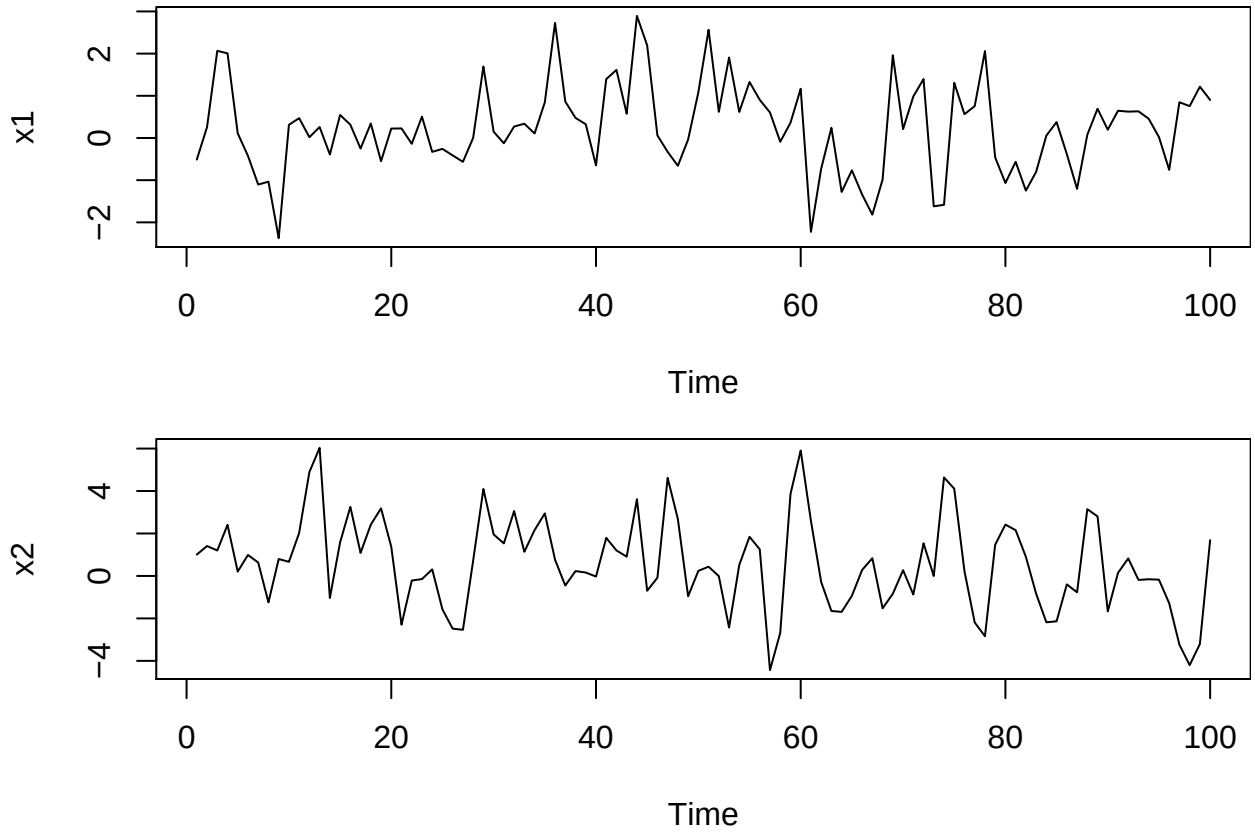
$$\rho(k) = \begin{cases} 1 & k = 0 \\ \frac{1/2}{1+(1/2)^2} = 2/5 & k = 1 \\ 0 & k > q \end{cases}$$

- For model 2 with $\beta_1 = 2$ we have

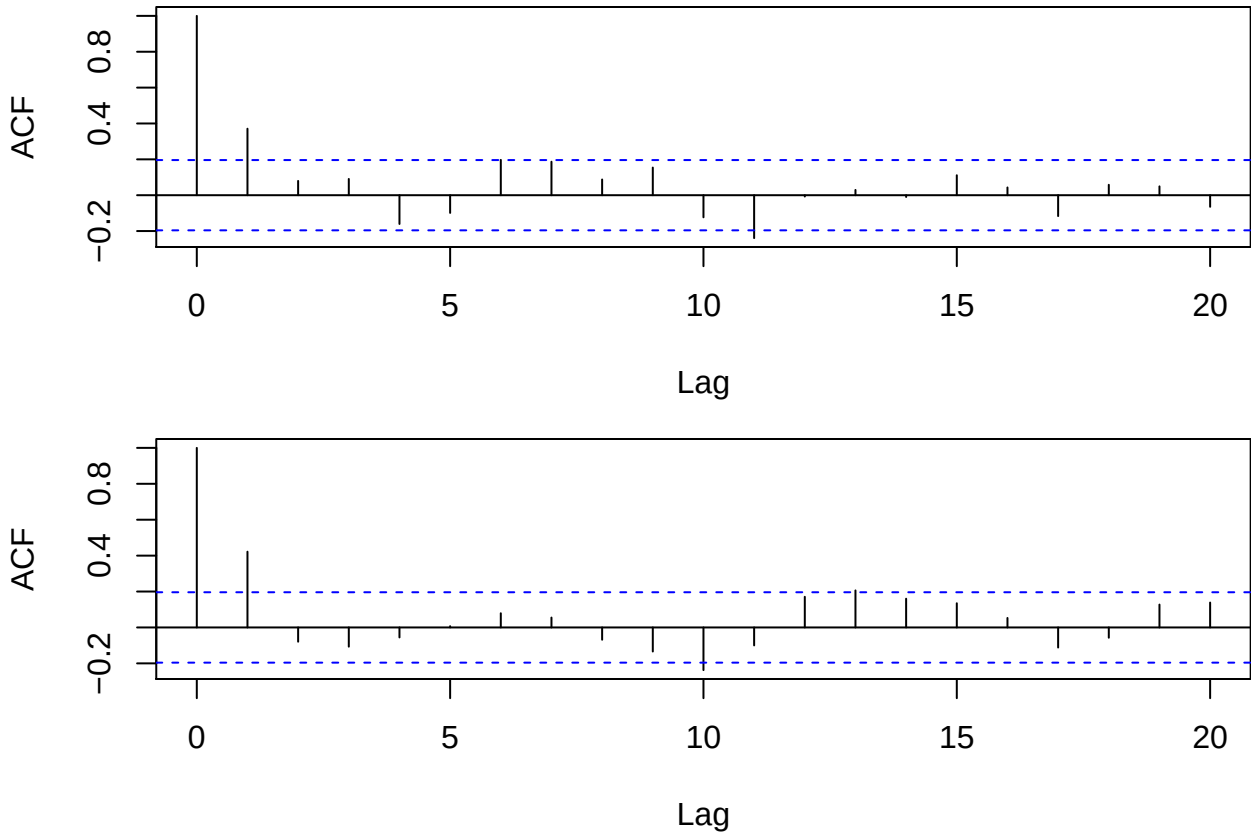
$$\rho(k) = \begin{cases} 1 & k = 0 \\ \frac{2}{1+2^2} = 2/5 & k = 1 \\ 0 & k > q \end{cases}$$

Data plot and empirical ACF

```
n <- 100
x1 <- arima.sim(list(ma=0.5), n = n)
x2 <- arima.sim(list(ma=2), n = n)
par(mfrow = c(2,1), mar = c(5,4,0,0))
plot(x1)
plot(x2)
```




```
par(mfrow = c(2,1), mar = c(5,4,0,0))
acf(x1)
acf(x2)
```



Variance

The constant theoretical variance is $\sigma^2 = \sigma_w^2(1 + \beta_1^2)$. In our simulations we didn't set the standard deviation of the white noise σ_w so R uses $\sigma_w = 1$ by default. Therefore, for

- model 1 ($\beta_1 = 1/2$) the theoretical variance is $1.25\sigma_w^2 = 1.25$
- model 2 ($\beta_1 = 2$) the theoretical variance is $5\sigma_w^2 = 5$

The observed variance for `x1` is 1.0716327 and the observed variance for `x2` is 4.499775.