

# Quality Control

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## 0.1 Outline

- Quality control
- Continuous process variable
- Binomial process variable
- Poisson process variable

# 1 Quality control

## 1.1 Quality control chart

Control charts are used to routinely monitor quality.

Two major types:

- **univariate control**: a graphical display (chart) of one quality characteristic
- **multivariate control**: a graphical display of a statistic that summarizes or represents more than one quality characteristic

The control chart shows

- the value of the quality characteristic versus the sample number or versus time
- a **center line** (CL) that represents the mean value for the in-control process
- an **upper control limit** (UCL) and a **lower control limit** (LCL)

The control limits are chosen so that almost all of the data points will fall within these limits **as long as the process remains in-control**.

## 1.2 Example

```
library(qcc)
data(pistonrings)
head(pistonrings,3)
```

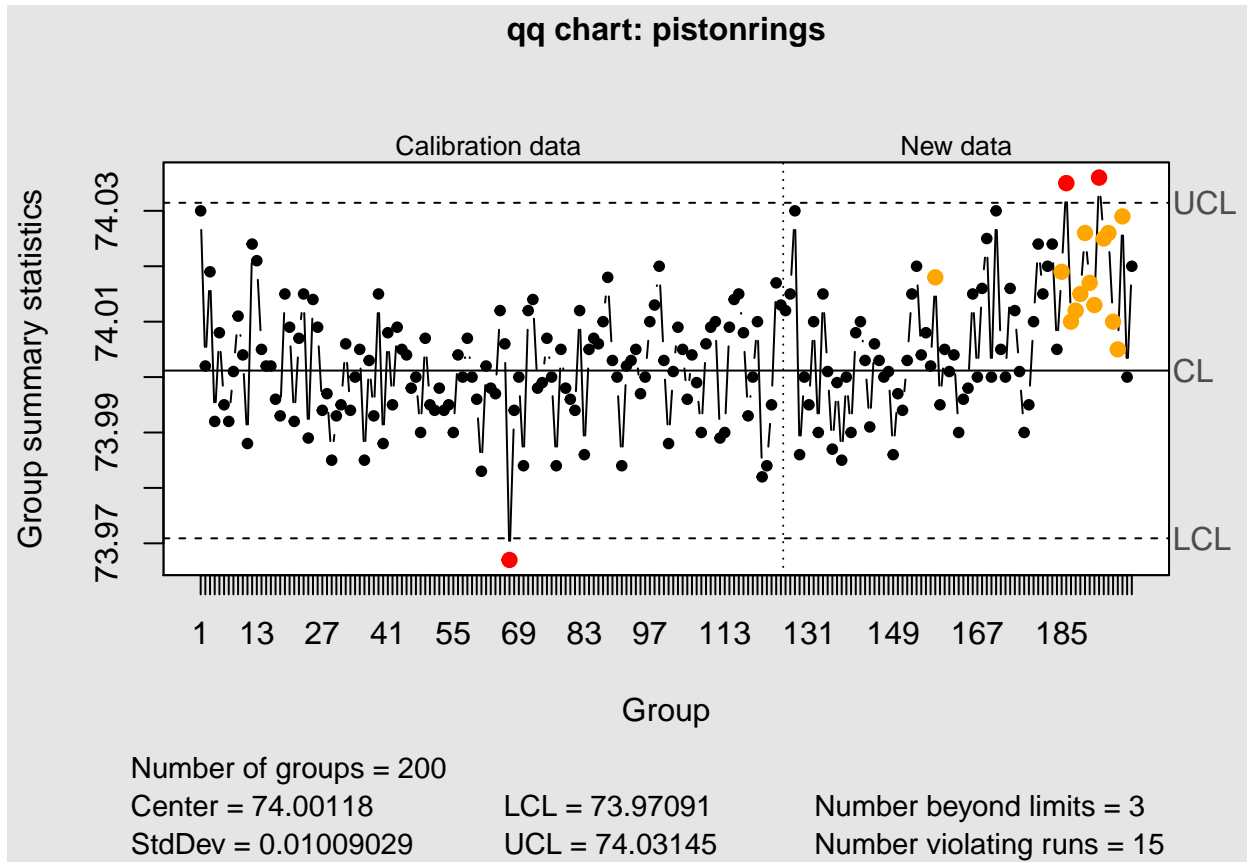
```
##   diameter sample trial
## 1    74.030      1  TRUE
## 2    74.002      1  TRUE
## 3    74.019      1  TRUE
```

Piston rings for an automotive engine are produced by a forging process. The inside diameter of the rings manufactured by the process is measured on 25 samples (`sample=1,2,...,25`), each of size 5, for the control phase I (`trial=TRUE`), when preliminary samples from a process being considered 'in-control' are used to construct control charts. Then, further 15 samples, again each of size 5, are obtained for phase II (`trial=FALSE`).

Reference:

Montgomery, D.C. (1991) Introduction to Statistical Quality Control, 2nd ed, New York, John Wiley & Sons, pp. 206-213

### 1.3 Example



We shall treat different methods for determining LCL,CL and UCL. In that respect, it is crucial that we have

- **phase I data**, where the process is in-control.
- These data are used to determine LCL,CL and UCL.

### 1.4 The simple six sigma model

Assume that measurements

- is a sample, i.e they are independent
- they have a normal distribution
- we know the mean  $\mu_0$  and standard deviation  $\sigma_0$ .

In this case we dont need phase I data.

- $CL = \mu_0$ .
- $LCL = \mu_0 - k\sigma_0$ .
- $UCL = \mu_0 + k\sigma_0$ .

The only parameter to determine is  $k$ .

We dont want to give a lot of false warnings, and a popular choice is

- $k=3$ , known as the 3\*sigma rule.
- The probability of a measurement outside the control limits is then 0.27%, when the proces is in-control.

This means that the span of allowable variation is  $6\sigma_0$ .

The concept “Six Sigma” has become a mantra in many industrial communities.

## 1.5 Average Run Length (ARL)

Let  $p_{out}$  denote the probability that a measurement is outside the control limits. On average this means that we need  $1/p_{out}$  observations before we get an outlier.

This is known as the *the Average Run Length*:

$$AVL = \frac{1}{p_{out}}$$

An in-control process with the 3\*sigma rule has AVL

```
round(1/(2*pnorm(-3, plot = FALSE)))
```

```
## [1] 370
```

An in-control process with AVL=500 has k\*sigma rule, where k equals

```
-qnorm(1/500)
```

```
## [1] 3.090232
```

## 1.6 Types of quality control charts.

Depending on the type of control variable, there are various types of control charts.

chart	distribution	statistic	example
xbar	normal	mean	means of a continuous process variable
S	normal	standard deviation	standard deviations of a continuous process variable
R	normal	range	ranges of a continuous process variable
p	binomial	proportion	percentage of faulty items
c	poisson	count	number of faulty items during a workday

## 2 Continuous process variable

### 2.1 Continuous process variable

Phase I data:

- $m$  samples with  $n$  measurements in each sample.
- For sample  $i = 1, 2, \dots, m$  calculate mean  $\bar{x}_i$  and standard deviation  $s_i$ .
- Calculate

$$\bar{\bar{x}} = \frac{1}{m} \sum_{i=1}^m \bar{x}_i \quad \text{and} \quad \bar{s} = \frac{1}{m} \sum_{i=1}^m s_i$$

When the sample is normal, it can be shown that  $\bar{s}$  is a biased estimate of the true standard deviation  $\sigma$ :

- $E(\bar{s}) = c_4(n)\sigma$
- $c_4(n)$  is tabulated in textbooks and available in the `qcc` package.

Unbiased estimate of  $\sigma$ :

$$\hat{\sigma}_1 = \frac{\bar{s}}{c_4(n)}$$

Furthermore  $\bar{s}$  has estimated standard error

$$se(\bar{s}) = \bar{s} \frac{\sqrt{1 - c_4(n)^2}}{c_4(n)}$$

### 2.2 xbar chart

$$\text{UCL: } \bar{\bar{x}} + 3 \frac{\hat{\sigma}_1}{\sqrt{n}}$$

$$\text{CL: } \bar{\bar{x}}$$

$$\text{LCL: } \bar{\bar{x}} - 3 \frac{\hat{\sigma}_1}{\sqrt{n}}$$

This corresponds to

- The probability of a measurement outside the control limits is 0.27%.

If we want to change this probability, we need another z-score. E.g if we want to lower this probability to 0.1%, then 3 should be substituted by 3.29.

### 2.3 Example

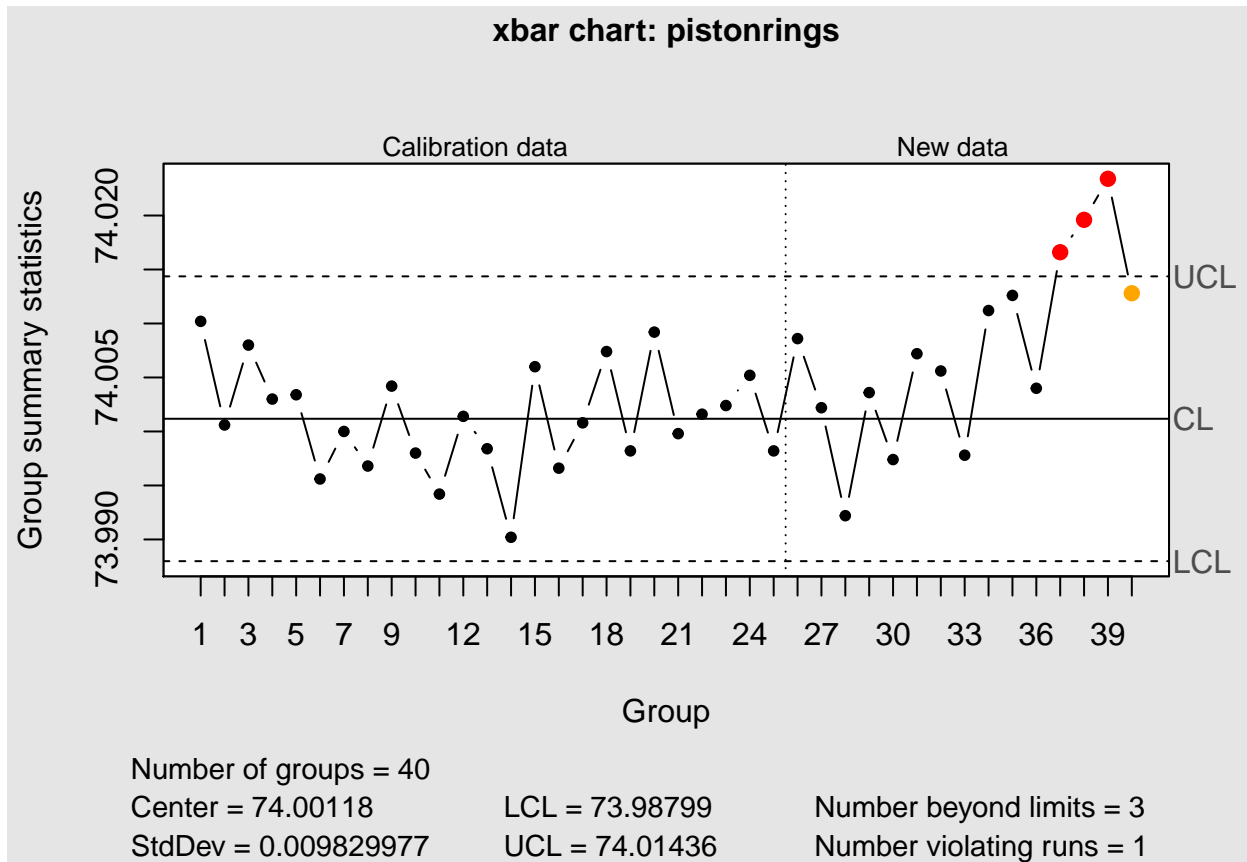
```
phaseI <- matrix(pistonrings$diameter[1:125] , nrow=25, byrow=TRUE)
phaseII <- matrix(pistonrings$diameter[126:200], nrow=25, byrow=TRUE)
h <- qcc(phaseI, type = "xbar", std.dev = "UWAVE-SD",
         newdata = phaseII, title = "xbar chart: pistonrings")
```

- `phaseI` is a matrix with  $m = 25$  rows, where each row is a sample of size  $n = 5$ .
- Similarly `phaseII` has 15 samples.

The function `qcc` calculates the necessary statistics and optionally makes a plot.

- `phaseI` and `type=` are the only arguments required.
- We want that the limits are based on the unweighted average of standard deviations - `UWAVE-SD`. This is not the default.
- We also want to evaluate the phase II data: `newdata=phaseII`.
- Optionally, we can specify the title on the plot.

## 2.4 Example



Besides limits we are also told whether the process is above/below CL for 7 or more consecutive samples (yellow dots).

`run.length=7` is default, but may be changed. If we e.g. want this to happen with probability 0.2%, then we specify `run.length=10`.

## 2.5 S chart: Monitoring variability

In most situations, it is crucial to monitor the process mean.

But it may also be a problem if the variability in “quality” gets too high.

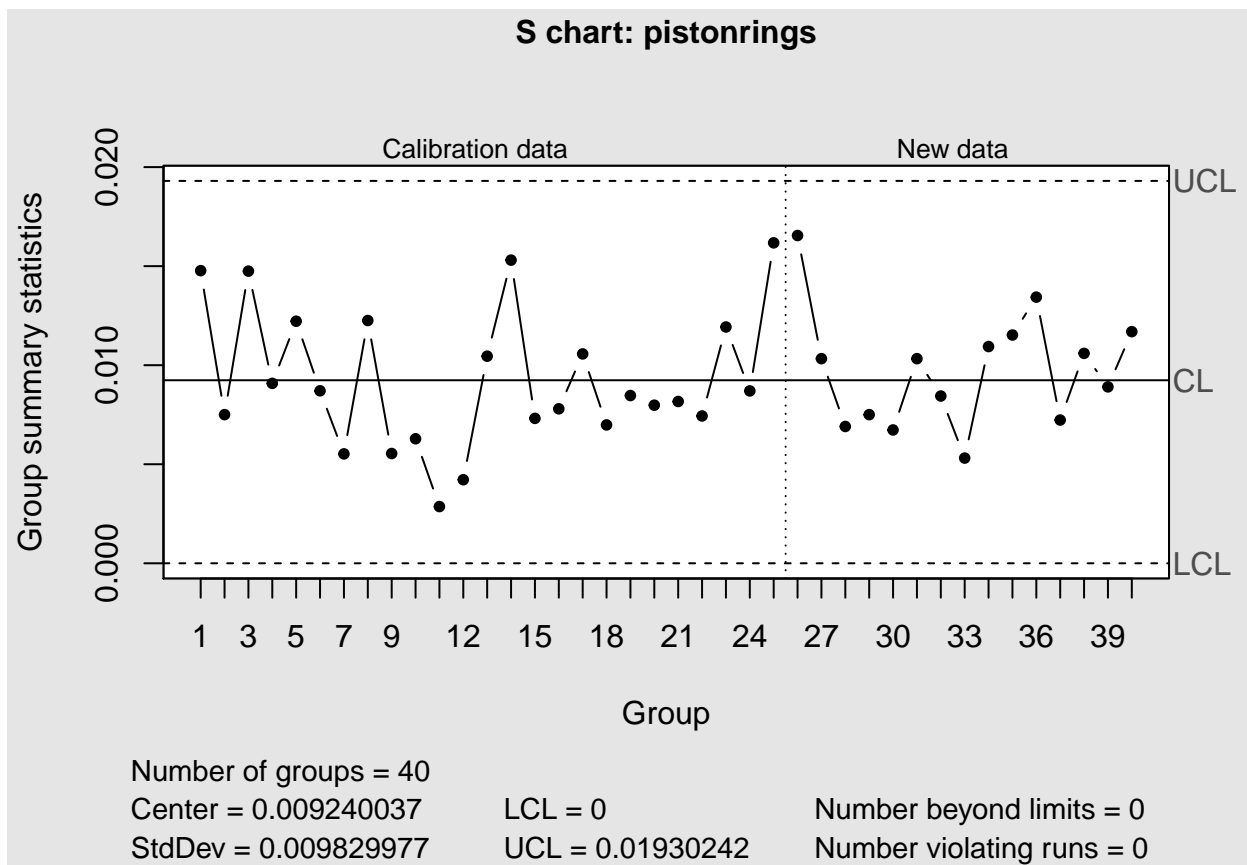
In that respect, it is relevant to monitor the standard deviation, which is done by the S-chart:

$$\begin{aligned} \text{UCL: } & \bar{s} + 3se(\bar{s}) \\ \text{CL: } & \bar{s} \\ \text{LCL: } & \bar{s} - 3se(\bar{s}) \\ se(\bar{s}) &= \bar{s} \frac{\sqrt{1 - c_4(n)^2}}{c_4(n)} \end{aligned}$$

Where 3 may be substituted by some other z-score depending on the required confidence level.

```
h <- qcc(phaseI,type="S", newdata=phaseII, title="S chart: pistonrings")
```

## 2.6 S chart example



Remark that the plot does not allow values below zero.

Quite sensible when we are talking about standard deviations.

## 2.7 R chart: Range statistics

If the sample size is relatively small ( $n \leq 10$ ), it is custom to use the range  $R$  instead of the standard deviation. The range of a sample is simply the difference between the largest and smallest observation.

When the sample is normal, it can be shown that:

- $E(\bar{R}) = d_2(n)\sigma$ , where  $\bar{R}$  is the average of the  $m$  sample ranges.

- $d_2(n)$  is tabulated in textbooks and available in the `qcc` package.

Unbiased estimate of  $\sigma$ :

$$\hat{\sigma}_2 = \frac{\bar{R}}{d_2(n)}$$

Furthermore  $\bar{R}$  has estimated standard error

$$se(\bar{R}) = \bar{R} \frac{d_3(n)}{d_2(n)}$$

$d_3(n)$  is tabulated in textbooks and available in the `qcc` package.

## 2.8 Charts based on $\bar{R}$

xbar chart based on  $\bar{R}$ :

$$\text{UCL: } \bar{x} + 3 \frac{\hat{\sigma}_2}{\sqrt{n}}$$

$$\text{CL: } \bar{x}$$

$$\text{LCL: } \bar{x} - 3 \frac{\hat{\sigma}_2}{\sqrt{n}}$$

This is actually the default in the `qcc` package.

R chart to monitor variability:

$$\text{UCL: } \bar{R} + 3se(\bar{R})$$

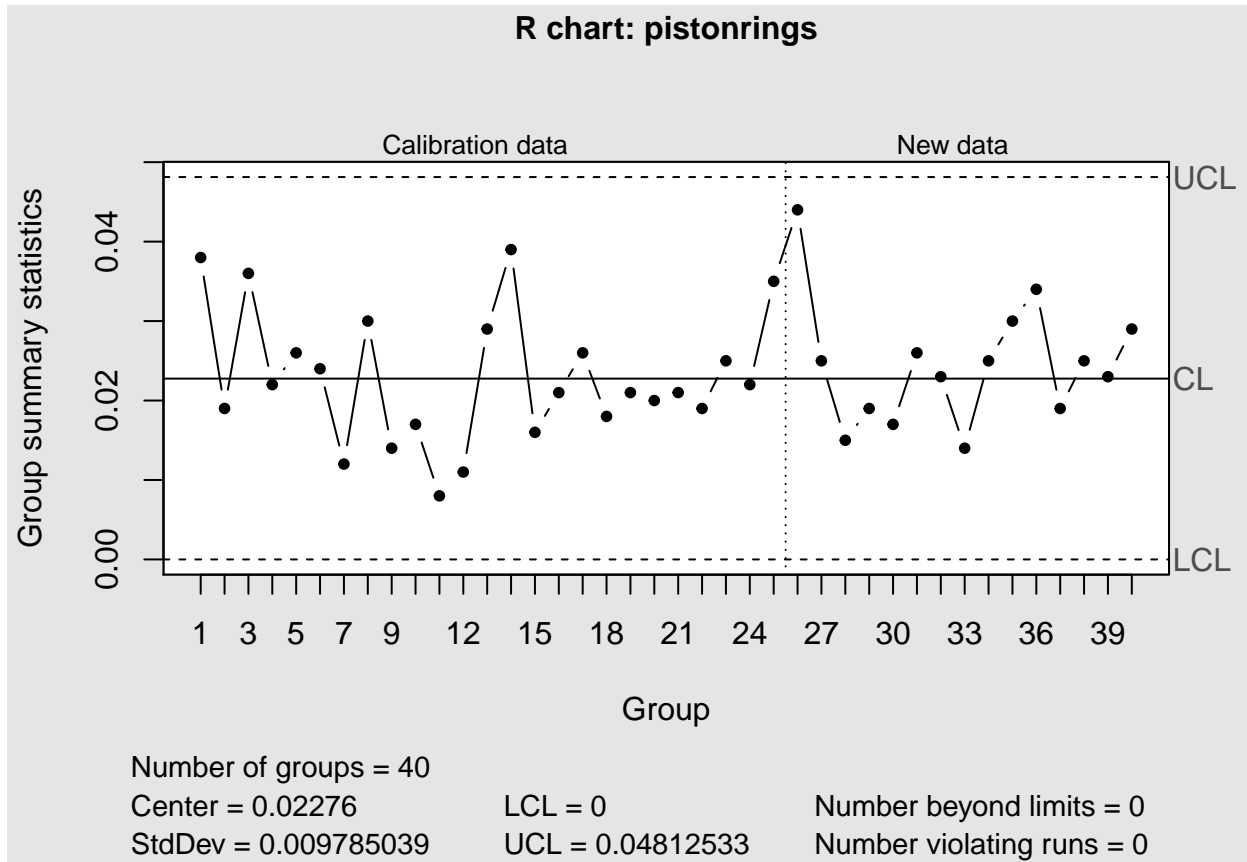
$$\text{CL: } \bar{R}$$

$$\text{LCL: } \bar{R} - 3se(\bar{R})$$

## 2.9 R chart example

```
h <- qcc(phaseI, type="R", newdata=phaseII, title="R chart: pistonrings")
```





### 3 Binomial process variable

#### 3.1 Binomial variation

Let us suppose that the production process operates in a stable manner such that

- the probability that an item is defect is  $p$ .
- successive items produced are independent

In a random sample of  $n$  items, the number  $D$  of defective items follows a binomial distribution with parameters  $n$  and  $p$ .

Unbiased estimate of  $p$ :

$$\hat{p} = \frac{D}{n}$$

which has standard error

$$se(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

#### 3.2 p chart

Data from phase I:

- $m$  samples with estimated proportions  $\hat{p}_i$ ,  $i = 1, \dots, m$

- $\bar{p}$  is the average of the estimated proportions.

p chart:

$$\begin{aligned} \text{UCL: } & \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\ \text{CL: } & \bar{p} \\ \text{LCL: } & \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \end{aligned}$$

### 3.3 Example

```
data(orangejuice)
head(orangejuice, 3)
```

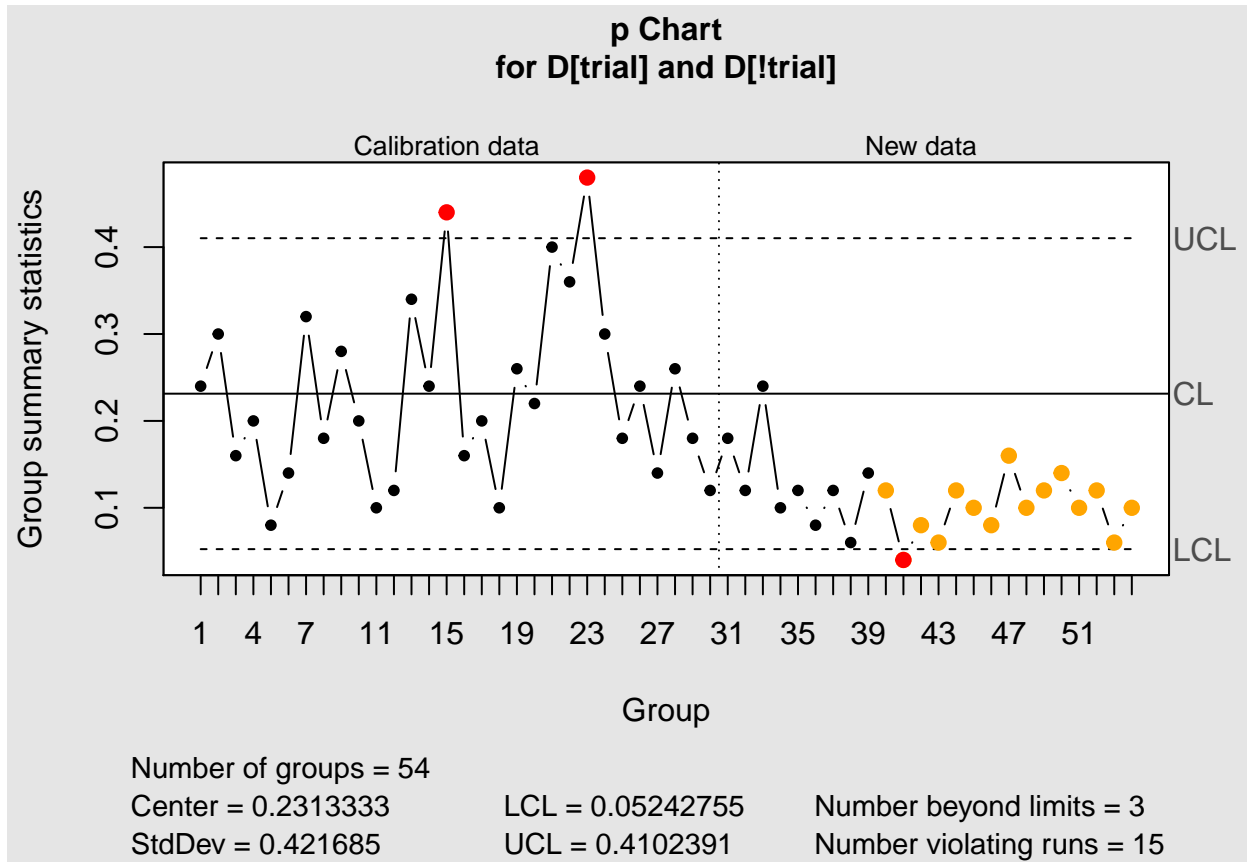
```
##  sample D size trial
## 1      1 12  50 TRUE
## 2      2 15  50 TRUE
## 3      3  8  50 TRUE
```

Production of orange juice cans.

- The data were collected as 30 samples of 50 cans.
- The number of defective cans D were observed.
- After the first 30 samples, a machine adjustment was made.
- Then further 24 samples were taken from the process.

```
with(orangejuice,
     qcc(D[trial], sizes=size[trial], type="p",
         newdata=D[!trial], newsizes=size[!trial]))
```

### 3.4 Example



The machine adjustment after sample 30 has had an obvious effect.

The chart should be recalibrated.

## 4 Poisson process variable

### 4.1 Poisson variation

Let us suppose that the production process operates in a stable manner such that

- defective items are produced at a constant rate

The number  $D$  of defective items over a time interval of some fixed length follows a poisson distribution with mean value  $c$ .

Unbiased estimate of  $c$ :

$$\hat{c} = D$$

which has standard error

$$se(\hat{c}) = \sqrt{c}$$

## 4.2 c chart

Data from phase I:

- $m$  sampling periods with mean estimates  $\hat{c}_i$ ,  $i = 1, \dots, m$
- $\bar{c}$  is the average of the estimated means.

c chart:

$$\text{UCL: } \bar{c} + 3\sqrt{\bar{c}}$$

$$\text{CL: } \bar{c}$$

$$\text{LCL: } \bar{c} - 3\sqrt{\bar{c}}$$