Intro and descriptive statistics

The ASTA team

Contents

1 Software

1.1 Rstudio

- Make a folder on your computer where you want to keep files to use in **Rstudio**. **Do NOT use Danish characters æ, ø, å** in the folder name (or anywhere in the path to the folder).
- Set the working directory to this folder: `Session -> Set Working Directory -> Choose Directory` (shortcut: Ctrl+Shift+H).
- Make the change permanent by setting the default directory in: `Tools -> Global Options -> Choose Directory`.

1.2 R basics

- Ordinary calculations:

```
4.6 * (2 + 3)^4
```

```
## [1] 2875
```

- Make a (scalar) object and print it:

```
a <- 4
a
```

```
## [1] 4
```

- Make a (vector) object and print it:

```
b <- c(2, 5, 7)
b
```

```
## [1] 2 5 7
```

- Make a sequence of numbers and print it:

```
s <- 1:4
s
```

```
## [1] 1 2 3 4
```

- Note: A more flexible command for sequences:

```
s <- seq(1, 4, by = 1)
```

- **R** does elementwise calculations:

```
a * b
```

```
## [1]  8 20 28
```

```
a + b
```

```
## [1]  6  9 11
```

```
b ^ 2
```

```
## [1]  4 25 49
```

- Sum and product of elements:

```
sum(b)
```

```
## [1] 14
```

```
prod(b)
```

```
## [1] 70
```

1.3 R extensions

- The functionality of **R** can be extended through libraries or packages (much like plugins in browsers etc.). Some are installed by default in **R** and you just need to load them.
- To install a new package in **Rstudio** use the menu: `Tools -> Install Packages`
- You need to know the name of the package you want to install. You can also do it through a command:

```
install.packages("mosaic")
```

- When it is installed you can load it through the `library` command:

```
library(mosaic)
```

- This loads the `mosaic` package which has a lot of convenient functions for this course (we will get back to that later). It also prints a lot of info about functions that have been changed by the `mosaic` package, but you can safely ignore that.

1.4 R help

- You get help via `?<command>`:

```
?sum
```

- Use `tab` to make **Rstudio** guess what you have started typing.
- Search for help:

```
help.search("plot")
```

- You can find a cheat sheet with the **R** functions we use for this course here.
- Save your commands in a file for later usage:
  - Select history tab in top right pane in **Rstudio** .
  - Mark the commands you want to save.
  - Press `To Source` button.

2 Data

2.1 Data example

Data: Magazine Ads Readability

- Thirty magazines were ranked by educational level of their readers.
- Three magazines were **randomly** selected from each of the following groups:
  - Group 1: highest educational level
  - Group 2: medium educational level
  - Group 3: lowest educational level.
- Six advertisements were **randomly** selected from each of the following nine selected magazines:
  - Group 1: [1] Scientific American, [2] Fortune, [3] The New Yorker
  - Group 2: [4] Sports Illustrated, [5] Newsweek, [6] People
  - Group 3: [7] National Enquirer, [8] Grit, [9] True Confessions
- So, the data contains information about a total of 54 advertisements.

2.2 Data example (continued) - variables and format

- For each advertisement (54 cases), the data below were observed.
- **Variable names**:
  - WDS = number of words in advertisement
  - SEN = number of sentences in advertisement
  - 3SYL = number of 3+ syllable words in advertisement
  - MAG = magazine (1 through 9 as above)
  - GROUP = educational level (1 through 3 as above)
- Take a look at the data from within **Rstudio**:

```
magAds <- read.delim("https://asta.math.aau.dk/datasets?file=magazineAds.txt")
head(magAds)
```

```
##   WDS SEN X3SYL MAG GROUP
## 1 205   9    34   1     1
## 2 203  20    21   1     1
## 3 229  18    37   1     1
## 4 208  16    31   1     1
## 5 146   9    10   1     1
## 6 230  16    24   1     1
```

- Variable names are in the top row. They are not allowed to start with a digit, so an `X` has been prefixed in `X3SYL`.

2.3 Data types

2.3.1 Quantitative variables

- The measurements have numerical values.
- Quantative data often comes about in one of the following ways:
  - **Continuous variables**: measurements of e.g. waiting times in a queue, revenue, share prices, etc.
  - **Discrete variables**: counts of e.g. words in a text, hits on a webpage, number of arrivals to a queue in one hour, etc.
- Measurements like this have a well-defined scale and in **R** they are stored as the type **numeric**.
- It is important to be able to distinguish between discrete count variables and continuous variables, since this often determines how we describe the uncertainty of a measurement.

2.3.2 Categorical/qualitative variables

- The measurement is one of a set of given categories, e.g. sex (male/female), social status, satisfaction score (low/medium/high), etc.
- The measurement is usually stored (which is also recommended) as a **factor** in **R**. The possible categories are called **levels**. Example: the levels of the factor "sex" is male/female.
- Factors have two so-called scales:
  - **Nominal scale**: There is no natural ordering of the factor levels, e.g. sex and hair color.
  - **Ordinal scale**: There is a natural ordering of the factor levels, e.g. social status and satisfaction score. A factor in **R** can have a so-called **attribute** assigned, which tells if it is ordinal.

3 Population and sample

3.1 Aim of statistics

- Statistics is all about "saying something" about a population.
- Typically, this is done by taking a random sample from the population.
- The sample is then analysed and a statement about the population can be made.
- The process of making conclusions about a population from analysing a sample is called **statistical inference**.

3.2 Selecting randomly

- For the magazine data:
  - First we select **randomly** 3 magazines from each group.
  - Then we select **randomly** 6 ads from each magazine.
  - An important detail is that the selection is done completely at **random**, i.e.
    - each magazine within a group have an equal chance of being chosen and
    - each ad within a magazine have an equal chance of being chosen.
- In the following it is a fundamental requirement that the data collection respects this principle of randomness and in this case we use the term **sample**.
- More generally:
  - We have a **population** of objects.
  - We choose completely at random n of these objects, and from the jth object we get the measurement y_j, $j = 1, 2, \ldots, n$.
  - The measurements $y_1, y_2, \ldots, y_n$ are then called a **sample**.
- If we e.g. are measuring the water quality 4 times in a year then it is a bad idea to only collect data in fair weather. The chosen sampling time is not allowed to be influenced by something that might influence the measurement itself.

4 Variable grouping and frequency tables

4.1 Binning

- The function `cut` will divide the range of a numeric variable in a number of equally sized intervals, and record which interval each observation belongs to. E.g. for the variable `X3SYL` (the number of words with more than three syllables) in the magazine data:

```
# Before 'cutting':
magAds$X3SYL[1:5]
```

```
## [1] 34 21 37 31 10
```

```
# After 'cutting' into 4 intervals:
syll <- cut(magAds$X3SYL, 4)
syll[1:5]
```

```
## [1] (32.2,43]     (10.8,21.5]   (32.2,43]     (21.5,32.2]   (-0.043,10.8]
## Levels: (-0.043,10.8] (10.8,21.5] (21.5,32.2] (32.2,43]
```

- The result is a `factor` and the labels are the interval end points by default. Custom ones can be assigned through the `labels` argument:

```
labs <- c("few", "some", "many", "lots")
syll <- cut(magAds$X3SYL, 4, labels = labs) # NB: this overwrites the 'syll' defined above
syll[1:5]
```

```
## [1] lots some lots many few
## Levels: few some many lots
```

```
magAds$syll <- syll # Adding a new column to the dataset
```

4.2 Tables

- To summarize the results we can use the function `tally` from the `mosaic` package (remember the package **must be loaded** via `library(mosaic)` if you did not do so yet):

```
tally( ~ syll, data = magAds)
```

```
## syll
##  few some many lots
##   26   14   10    4
```

- In percent:

```
tally( ~ syll, data = magAds, format = "percent")
```

```
## syll
##  few some many lots
## 48.1 25.9 18.5  7.4
```

- Here we use an **R** `formula` (characterized by the "tilde" sign `~`) to indicate that we want this variable from the dataset `magAds` (without the tilde it would look for a global variable called `syll` and use that rather than the one in the dataset).

4.3 2 factors: Cross tabulation

- To make a table of all combinations of two factors we use `tally` again:

```
tally( ~ syll + GROUP, data = magAds)
```

```
##       GROUP
## syll    1  2  3
##   few   8 11  7
##   some  4  2  8
##   many  3  5  2
##   lots  3  0  1
```

- Relative frequencies (in percent) columnwise:

```
tally( ~ syll | GROUP, data = magAds, format = "percent")
```

```
##       GROUP
## syll      1    2    3
##   few  44.4 61.1 38.9
##   some 22.2 11.1 44.4
##   many 16.7 27.8 11.1
##   lots 16.7  0.0  5.6
```

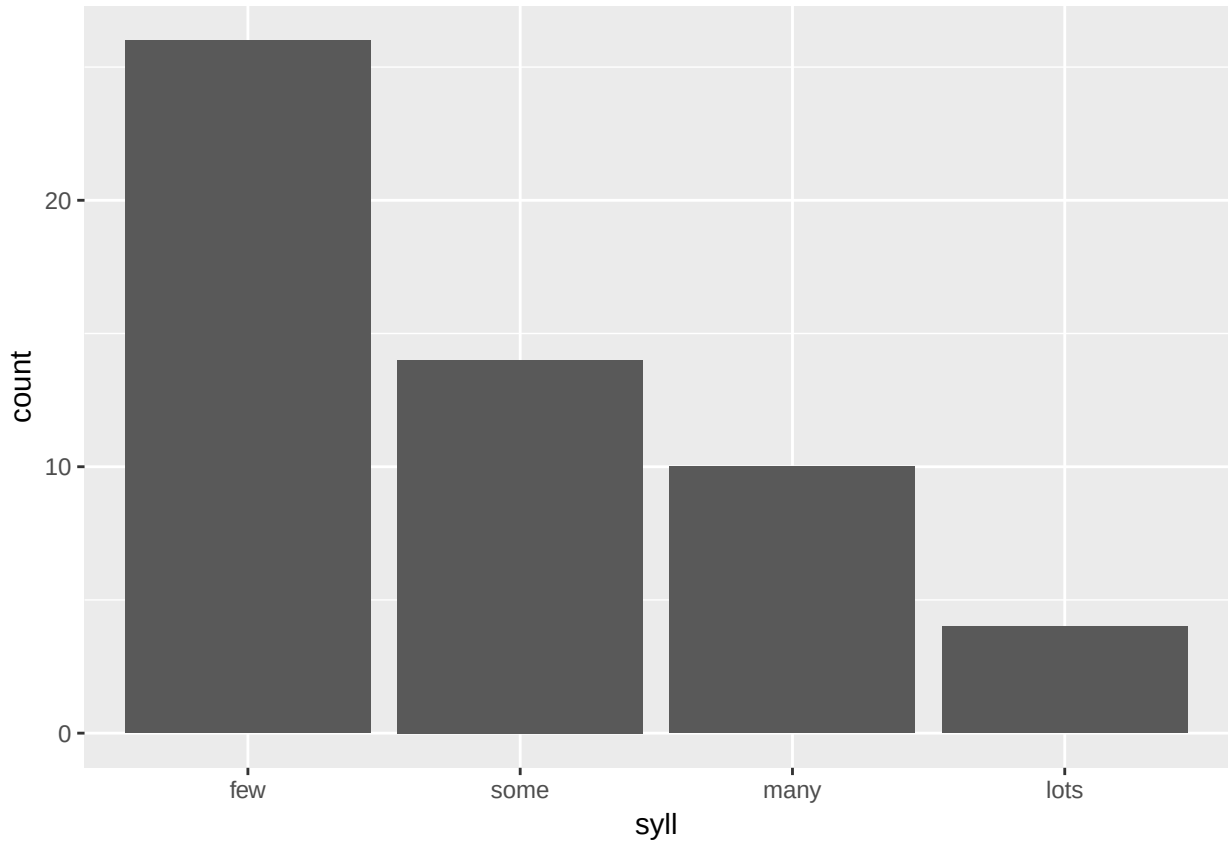
- So, the above table shows e.g. how many percentage of the advertisements in group 1 that have 'few', 'some', 'many' or 'lots' words with more than 3 syllables.

5 Graphics

5.1 Bar graph

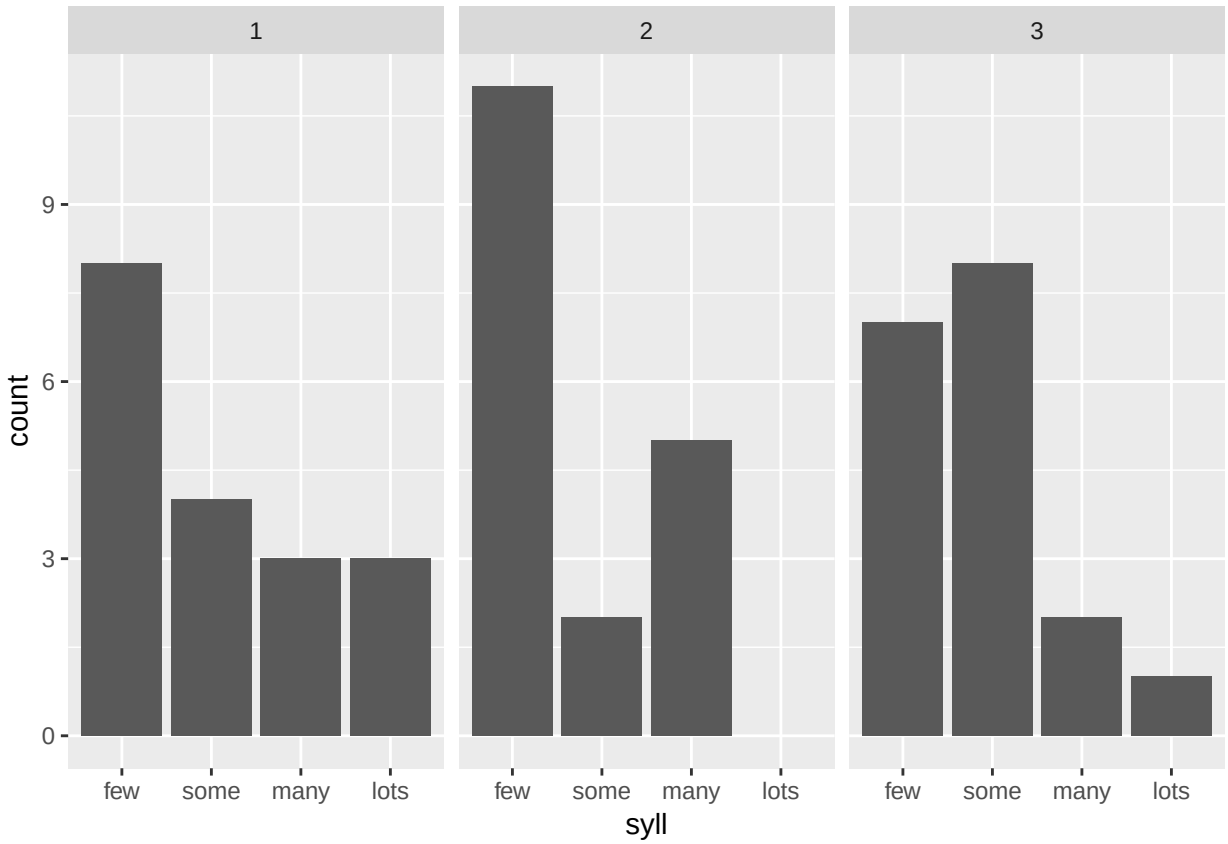
- To create a bar graph plot of table data we use the function `gf_bar` from `mosaic`. For each level of the factor a box is drawn with the height proportional to the frequency (count) of the level.

```
gf_bar( ~ syll, data = magAds)
```



- The bar graph can also be split by group:

```
gf_bar( ~ syll | GROUP, data = magAds)
```



5.2 The Ericksen data

- Description of data: Ericksen 1980 U.S. Census Undercount.
- This data contains the following variables:
  - `minority`: Percentage black or Hispanic.
  - `crime`: Rate of serious crimes per 1000 individuals in the population.
  - `poverty`: Percentage poor.
  - `language`: Percentage having difficulty speaking or writing English.
  - `highschool`: Percentage aged 25 or older who had not finished highschool.
  - `housing`: Percentage of housing in small, multiunit buildings.
  - `city`: A factor with levels: `city` (major city) and `state` (state or state-remainder).
  - `conventional`: Percentage of households counted by conventional personal enumeration.
  - `undercount`: Preliminary estimate of percentage undercount.
- The Ericksen data has 66 rows/observations and 9 columns/variables.
- The observations are measured in 16 large cities, the remaining parts of the states in which these cities are located, and the other U.S. states.

```
Ericksen <- read.delim("https://asta.math.aau.dk/datasets?file=Ericksen.txt")
head(Ericksen)
```

```
##          name minority crime poverty language highschool housing  city
## 1     Alabama     26.1    49      19      0.2         44     7.6 state
## 2      Alaska      5.7    62      11      1.7         18    23.6 state
## 3     Arizona     18.9    81      13      3.2         28     8.1 state
```

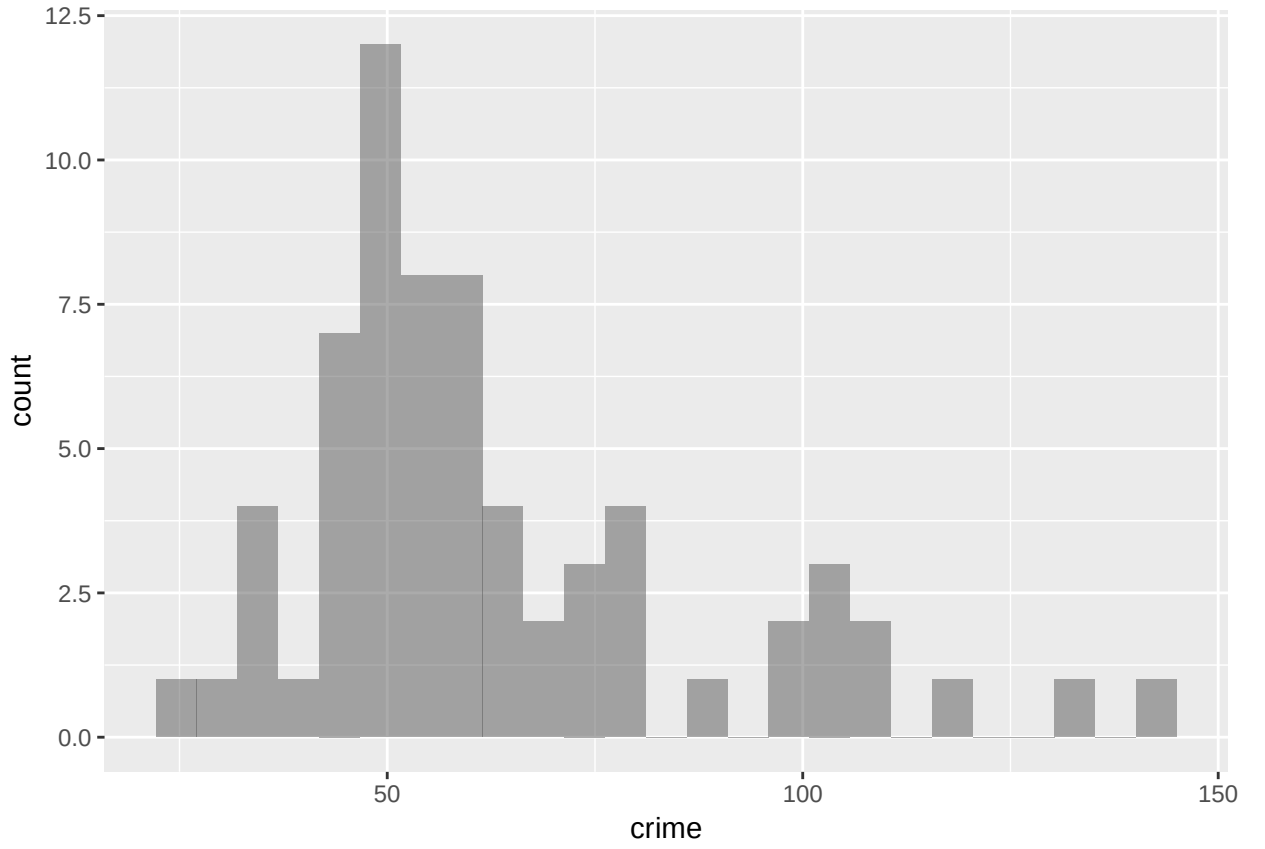
```
## 4     Arkansas     16.9    38      19      0.2         44     7.0 state
## 5 California.R     24.3    73      10      5.0         26    11.8 state
## 6     Colorado     15.2    73      10      1.2         21     9.2 state
##   conventional undercount
## 1            0      -0.04
## 2          100       3.35
## 3           18       2.48
## 4            0      -0.74
## 5            4       3.60
## 6           19       1.34
```

- Want to make a histogram for crime rate - how?

5.3 Histogram (quantitative variables)

- How to make a histogram for some variable `x`:
  - Divide the interval from the minimum value of `x` to the maximum value of `x` in an appropriate number of equal sized sub-intervals.
  - Draw a box over each sub-interval with the height being proportional to the number of observations in the sub-interval.
- Histogram of crime rates for the Ericksen data

```
gf_histogram( ~ crime, data = Ericksen)
```



6 Summary of quantitative variables

6.1 Measures of center of data: Mean and median

- We return to the magazine ads example (`WDS` = number of words in advertisement). A number of numerical summaries for `WDS` can be retrieved using the `favstats` function:

```
favstats( ~ WDS, data = magAds)
```

```
##  min Q1 median  Q3 max mean sd  n missing
##   31 69     96 202 230  123 66 54       0
```

- The observed values of the variable `WDS` are $y_1 = 205$, $y_2 = 203, \ldots, y_n = 208$, where there are a total of $n = 54$ values. As previously defined this constitutes a **sample**.
- **mean** $= 123$ is the **average** of the sample, which is calculated by

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

  We may also call $\bar{y}$ the **(empirical) mean** or the **sample mean**.
- **median** $= 96$ is the 50-percentile, i.e. the value that splits the sample in 2 groups of equal size.
- An important property of the **mean** and the **median** is that they have the same unit as the observations (e.g. meter).

6.2 Measures of variability of data: range, standard deviation and variance

- The **range** is the difference of the largest and smallest observation.
- The **(empirical) variance** is the average of the squared deviations from the mean:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2.$$

- **sd** $=$ **standard deviation** $= s = \sqrt{s^2}$.
- Note: If the observations are measured in meter, the **variance** has unit meter^2 which is hard to interpret. The **standard deviation** on the other hand has the same unit as the observations.
- The standard deviation describes how much data varies around the (empirical) mean.

6.3 Calculation of mean, median and standard deviation using R

The mean, median and standard deviation are just some of the summaries that can be read of the `favstats` output (shown on previous page). They may also be calculated separately in the following way:

- Mean of `WDS`:

```
mean( ~ WDS, data = magAds)
```

```
## [1] 123
```

- Median of `WDS`:

```
median( ~ WDS, data = magAds)
```

```
## [1] 96
```

- Standard deviation for `WDS`:

```
sd( ~ WDS, data = magAds)
```

```
## [1] 66
```

We may also calculate the summaries for each group (variable `GROUP`), e.g. for the mean:

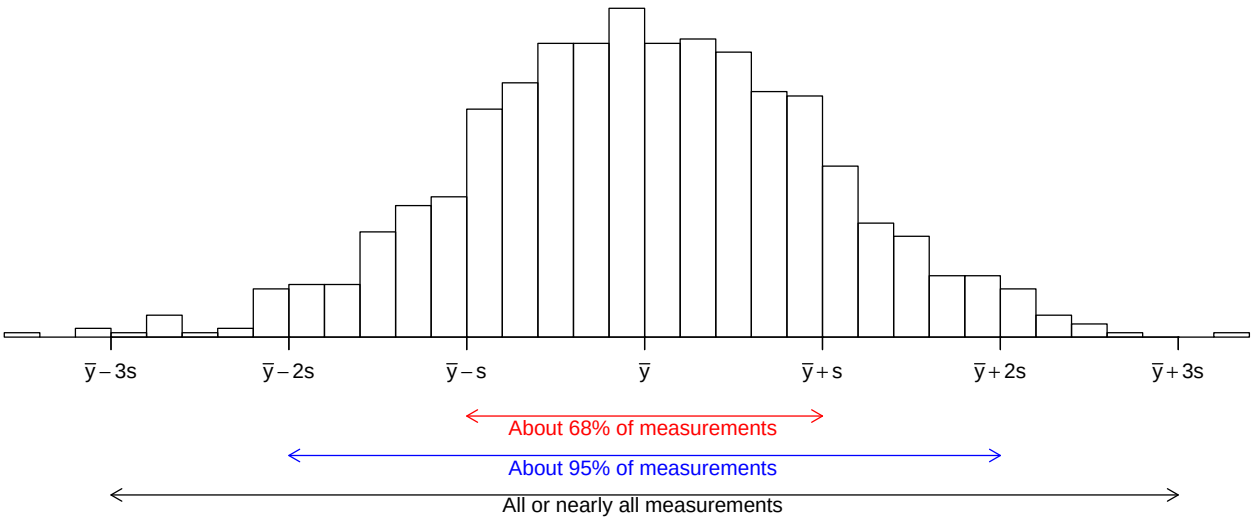
```
mean( ~ WDS | GROUP, data = magAds)
```

```
##   1   2   3
## 140 121 106
```

6.4 A word about terminology

- **Standard deviation**: a measure of variability of a population or a sample.
- **Standard error**: a measure of variability of an estimate. For example, a measure of variability of the sample mean.

6.5 The empirical rule



If the histogram of the sample looks like a bell shaped curve, then

- about 68% of the observations lie between $\bar{y} - s$ and $\bar{y} + s$.
- about 95% of the observations lie between $\bar{y} - 2s$ and $\bar{y} + 2s$.
- All or almost all (99.7%) of the observations lie between $\bar{y} - 3s$ and $\bar{y} + 3s$.

6.6 Percentiles

- **The pth percentile** is a value such that about $p\%$ of the population (or sample) lies below or at this value and about $(100 - p)\%$ of the population (or sample) lies above it.

6.6.1 Percentile calculation for a sample:

- First, sort data in increasing order. For the `WDS` variable in the magazine data:

$$y_{(1)} = 31, y_{(2)} = 32, y_{(3)} = 34, \ldots, y_{(n)} = 230.$$

  Here the number of observations is $n = 54$.

- Find the 5th percentile (i. e. $p = 5$):
  - The observation number corresponding to the 5-percentile is $N = \frac{54 \cdot 5}{100} = 2.7$.
  - So the 5-percentile lies between the observations with observation number $k = 2$ and $k + 1 = 3$. That is, its value lies somewhere in the interval between $y_2 = 32$ and $y_3 = 34$
  - One of several methods for estimating the 5-percentile from the value of N is defined as:

$$y_{(k)} + (N - k)(y_{(k+1)} - y_{(k)})$$

    which in this case states

$$y_2 + (2.7 - 2)(y_3 - y_2) = 32 + 0.7 \cdot (34 - 32) = 33.4$$

6.7 Median, quartiles and interquartile range

Recall

```
favstats( ~ WDS, data = magAds)
```

```
##  min Q1 median  Q3 max mean sd  n missing
##   31 69     96 202 230  123 66 54       0
```

- 50-percentile = 96 is the **median** and it is a measure of the center of data.
- 0-percentile = 31 is the **minimum** value.
- 25-percentile = 69 is called the **lower quartile** (Q1). Median of lower 50% of data.
- 75-percentile = 202 is called the **upper quartile** (Q3). Median of upper 50% of data.
- 100-percentile = 230 is the **maximum** value.
- **Interquartile Range (IQR)**: a measure of variability given by the difference of the upper and lower quartiles: 202 - 69 = 133.

7 More graphics

7.1 Box-and-whiskers plots (or simply box plots)

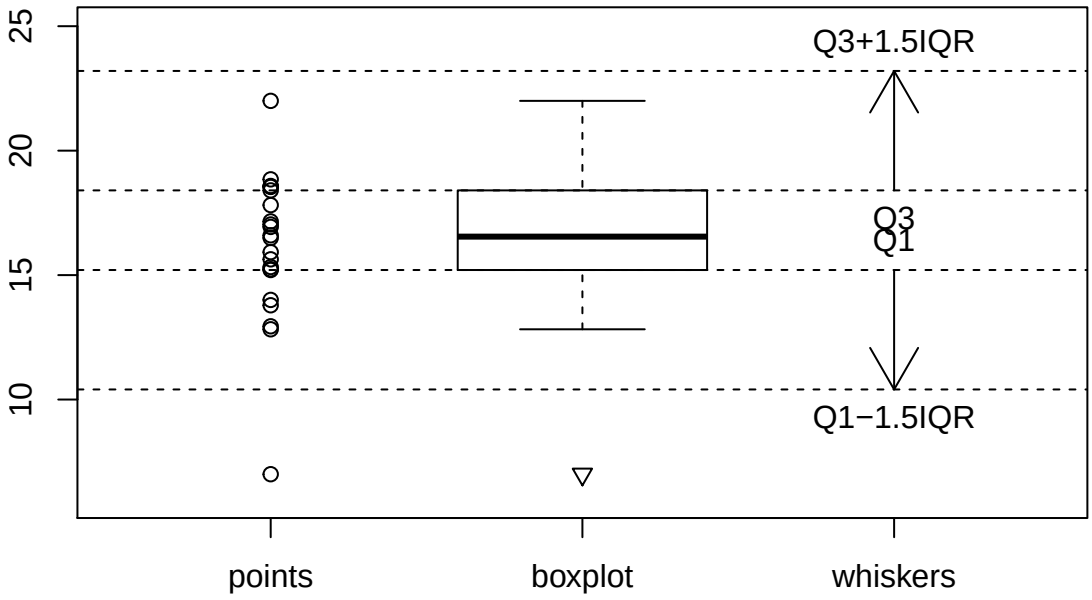
How to draw a box-and-whiskers plot:

- Box:
  - Calculate the median, lower and upper quartiles.

  - Plot a line by the median and draw a box between the upper and lower quartiles.

- Whiskers:
  - Calculate interquartile range and call it IQR.
  - Calculate the following values:
    * L = lower quartile - 1.5*IQR
    * U = upper quartile + 1.5*IQR
  - Draw a line from lower quartile to the smallest measurement, which is larger than L.
  - Similarly, draw a line from upper quartile to the largest measurement which is smaller than U.
- Outliers: Measurements smaller than L or larger than U are drawn as circles.

Note: Whiskers are minimum and maximum of the observations that are not deemed to be outliers.



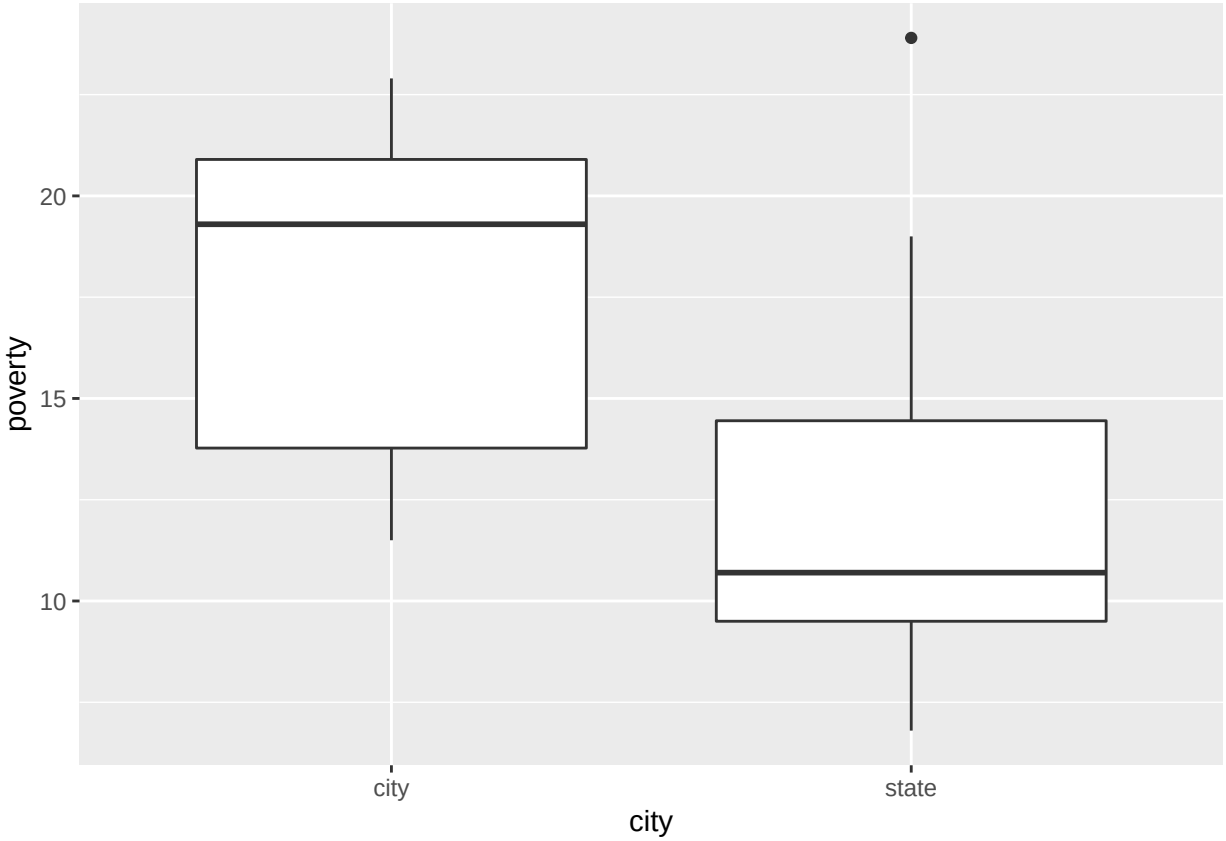
7.1.1 Boxplot for Ericksen data

Boxplot of the poverty rates separately for cities and states (variable `city`):

```
favstats(poverty ~ city, data = Ericksen)
```

```
##    city  min   Q1 median Q3 max mean  sd  n missing
## 1  city 11.5 13.8     19 21  23   18 4.0 16       0
## 2 state  6.8  9.5     11 14  24   12 3.7 50       0
```

```
gf_boxplot(poverty ~ city, data = Ericksen)
```

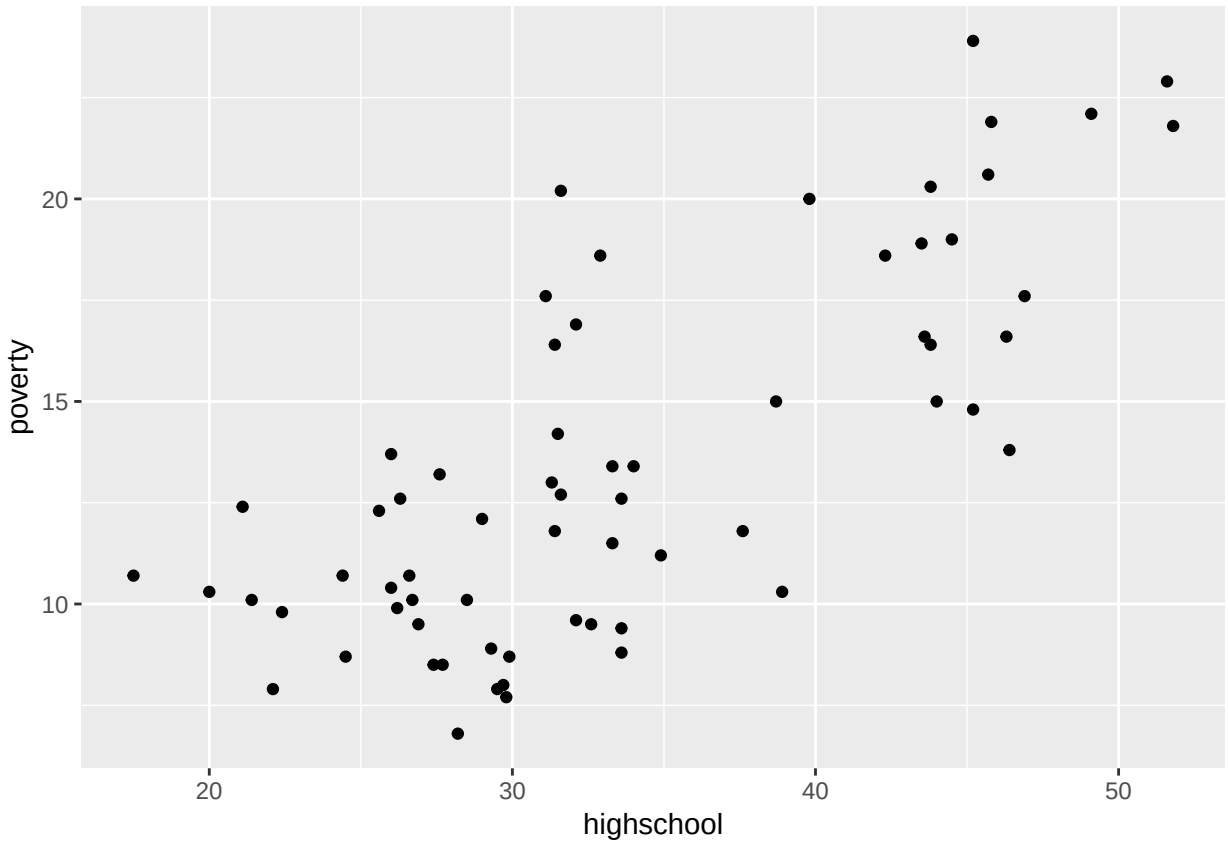


- There seems to be more poverty in the cities.
- A single state differs noticeably from the others with a high poverty rate.

7.2 2 quantitative variables: Scatter plot

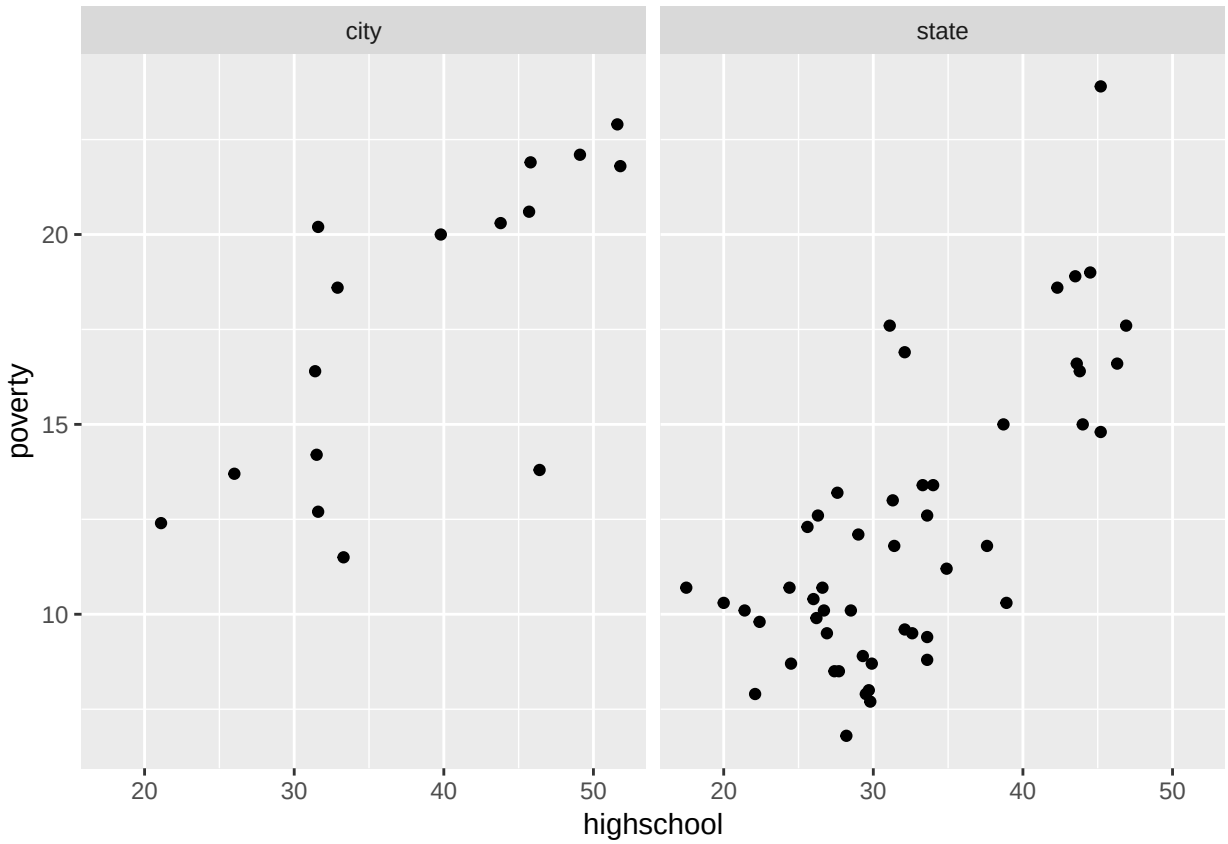
For two quantitative variables the usual graphic is a scatter plot:

```
gf_point(poverty ~ highschool, data = Ericksen)
```

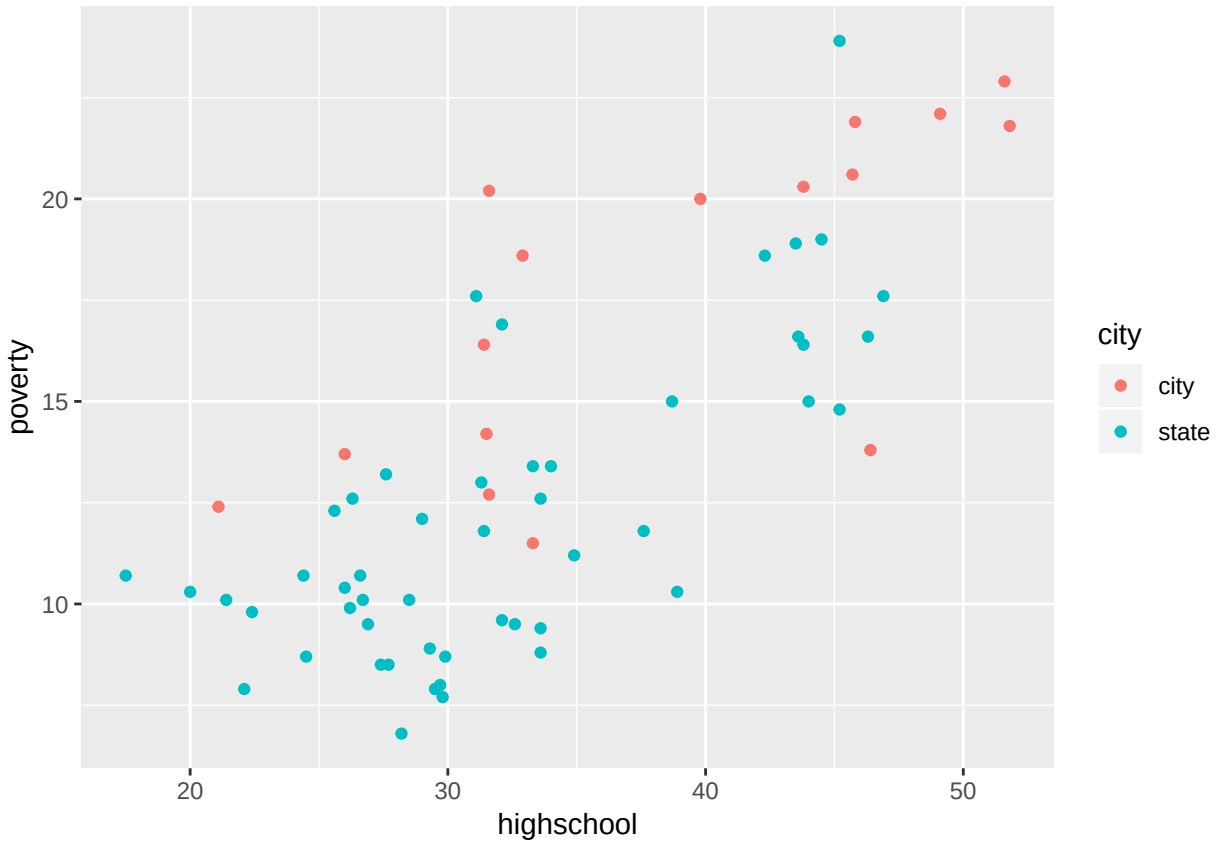


This can be either split or coloured according to the value of `city`:

```
gf_point(poverty ~ highschool | city, data = Ericksen)
```

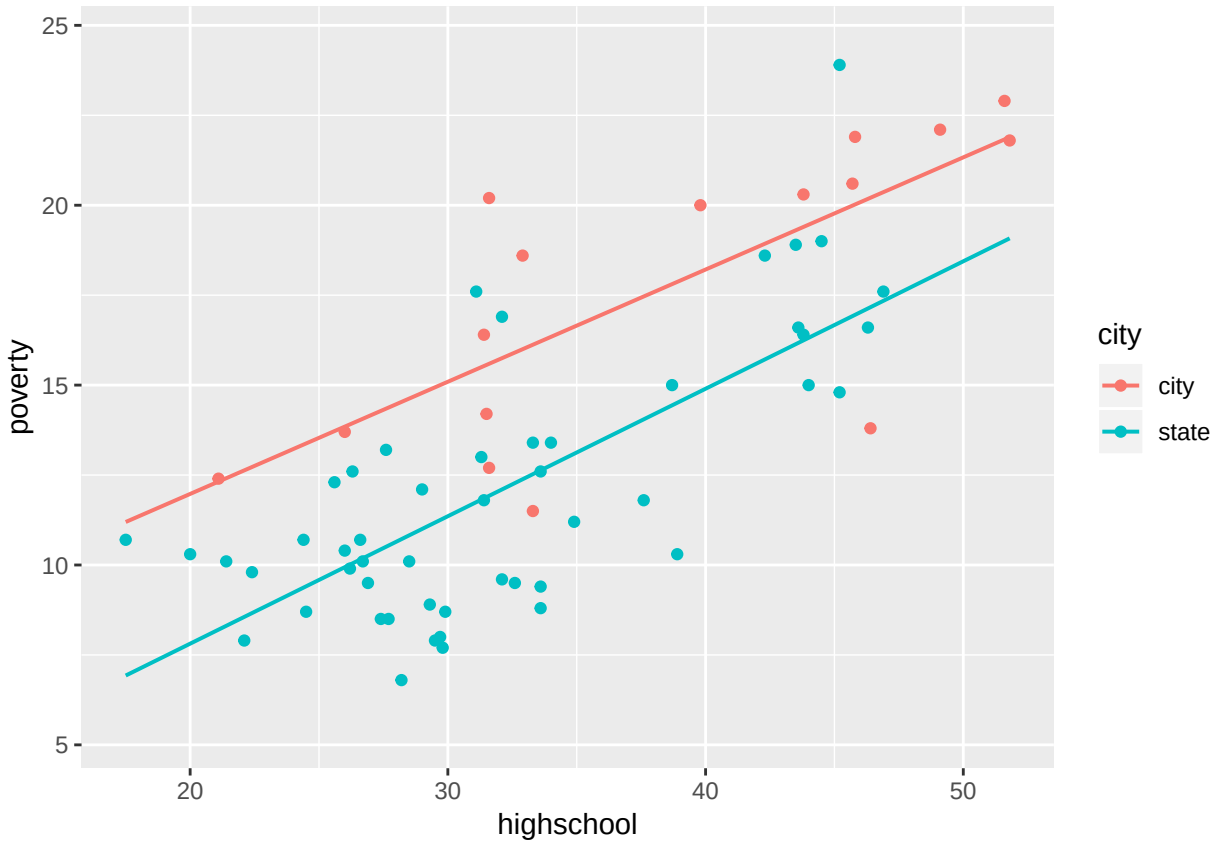



```
gf_point(poverty ~ highschool, col = ~city, data = Ericksen)
```



If we want a regression line along with the points we can do:

```
gf_point(poverty ~ highschool, col = ~city, data = Ericksen) %>% gf_lm()
```



8 Appendix

8.1 Recoding variables

- The function `factor` will directly convert a vector to be of type `factor`. E.g.:

```
head(magAds$GROUP)
```

```
## [1] 1 1 1 1 1 1
```

```
f <- factor(magAds$GROUP)
magAds$GROUP <- f
head(magAds$GROUP)
```

```
## [1] 1 1 1 1 1 1
## Levels: 1 2 3
```

- Custom labels for the levels can also be used:

```
f <- factor(magAds$GROUP,
            levels = c("1", "2", "3"),
            labels = c("high", "medium", "low"))
magAds$GROUP <- f
head(magAds$GROUP)
```

```
## [1] high high high high high high
## Levels: high medium low
```

- In this way the numbers are replaced by more informative labels describing the educational level.