Ordinal data

The ASTA team

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1 Association between ordinal variables

1.1 Ordinal variables

For a random sample of black males the General Social Survey in 1996 asked two questions:

- Q1: What is your yearly income (`income`)?
- Q2: How satisfied are you with your job (`satisfaction`)?

Both measurements are on an ordinal scale.

	VeryD	LittleD	ModerateS	VeryS
< 15k	1	3	10	6
15-25k	2	3	10	7
25-40k	1	6	14	12
> 40k	0	1	9	11

We might do a chi-square test to see whether Q1 and Q2 are associated, but the test does not exploit the ordinality.

We shall consider a test that incorporates ordinality.

1.2 Gamma coefficient

Consider a pair of respondents, where respondent1 is below respondent2 in relation to Q1.

- If respondent1 is also below respondent2 in relation to Q2 then the pair is *concordant*.
- If respondent1 is above respondent2 in relation to Q2 then the pair is *disconcordant*.

Let

- C = the number of concordant pairs in our sample.

- $D =$ the number of disconcordant pairs in our sample.

We define the estimated *gamma coefficient*

$$\hat{\gamma} = \frac{C-D}{C+D} = \underbrace{\frac{C}{C+D}}_{concordant\ prop.} - \underbrace{\frac{D}{C+D}}_{discordant\ prop.}$$

1.3 Gamma coefficient

Properties:

- Gamma lies between -1 og 1
- The sign tells whether the association is positive or negative
- Large absolute values correspond to strong association

The standard error $se(\hat{\gamma})$ on $\hat{\gamma}$ is complicated to calculate, so we leave that to software.

We can now determine a 95% confidence interval:

$$\hat{\gamma} \pm 1.96 se(\hat{\gamma})$$

and if zero is contained in the interval, then there is no significant association, when we perform a test with a 5% significance level.

1.4 Example

First, we need to install the package `vcdExtra`, which has the function `GKgamma` for calculating gamma. It also has the dataset on job satisfaction and income built-in:

```
library(vcdExtra)
JobSat
```

```
##         satisfaction
## income   VeryD LittleD ModerateS VeryS
##   < 15k      1       3        10     6
##   15-25k     2       3        10     7
##   25-40k     1       6        14    12
##   > 40k      0       1         9    11
```

```
GKgamma(JobSat, level = 0.90)
```

```
## gamma        : 0.221
## std. error   : 0.117
## CI           : 0.028 0.414
```

A positive association. Marginally significant at the 10% level, but not so at the 5% level.